

Polarized Neutron Scattering

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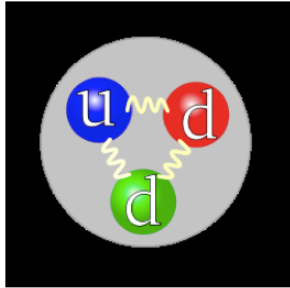
SwedNess/NNSP, Tartu Estonia, September 18th 2019

Repetition / Basics of

neutron properties

magnetic neutron scattering

The neutron



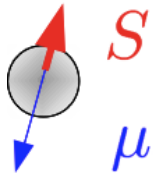
Source: Wikipedia

Charge 0
Spin 1/2

Quarks

	Charge	Spin
u	2/3	1/2
d	-1/3	1/2

Neutron is a **spin 1/2 particle**, the spin is tied to a **magnetic moment**.



$$\mu_n = g_n \cdot S \cdot \mu_N \simeq \mp 1.913 \mu_N = \pm \gamma_n \mu_N.$$

Note the gyromagnetic factor $\gamma_n = -1.913$ for the neutron is negative, *i.e.* spin and magnetic moment are antiparallel

$$\mu_N \equiv \frac{e\hbar}{2m_p}$$

$$\frac{\mu_N}{\mu_B} = \frac{m_e}{m_n} \approx \frac{1}{1836}$$

neutron interacts with nuclei

Its **spin** interacts with spin of nuclei

Its **magnetic moment** interacts with magnetic moments of unpaired electrons

magnetic scattering

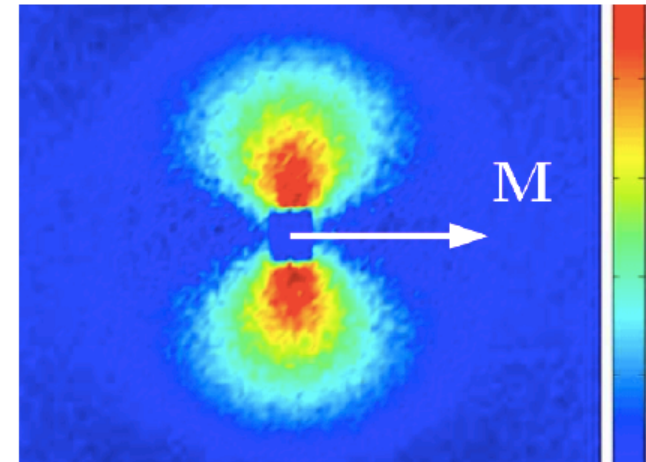
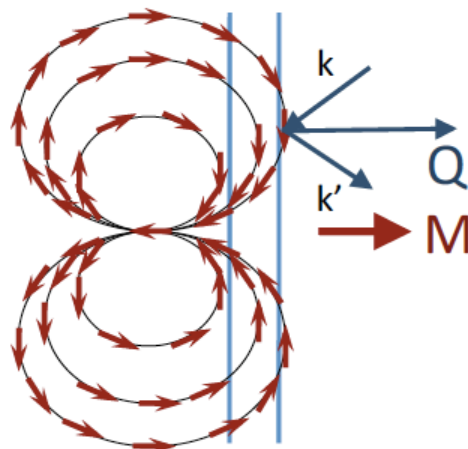
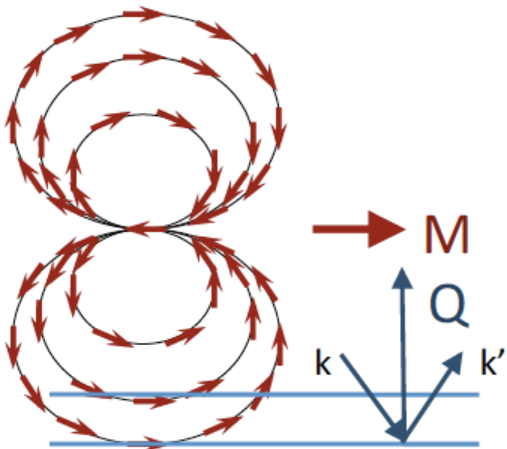
Neutron spins

dipole-dipole interaction with magnetic fields of unpaired electrons

$$V_m = -\mu_{(n)} \cdot (\mathbf{B}_S + \mathbf{B}_L)$$

$$V_m = -(\gamma_n r_0 / 2) \hat{\sigma} \cdot \hat{\mathbf{M}}_{\mathbf{Q}}^{\perp}$$

$$\mathbf{M}_{\mathbf{Q}}^{\perp} = \mathbf{e}_{\mathbf{Q}} \times \mathbf{M}_{\mathbf{Q}} \times \mathbf{e}_{\mathbf{Q}}$$



we can only see the moments perpendicular to \mathbf{Q} !

magnetic scattering

Neutron spins

dipole-dipole interaction with magnetic fields of unpaired electrons

$$V_m = -\mu_{(n)} \cdot (\mathbf{B}_S + \mathbf{B}_L)$$

$$V_m = -(\gamma_n r_0 / 2) \hat{\sigma} \cdot \hat{\mathbf{M}}_Q^\perp$$

Magnetic scattering amplitudes (Bacon 1975)

$$p = -0.27 \times 10^{-12} \text{cm} \cdot \langle \mu \rangle f(Q)$$

neutron electron coupling constant

magnetic moment
in units μ_B

magnetic formfactor –
Fourier transform of
the spin density

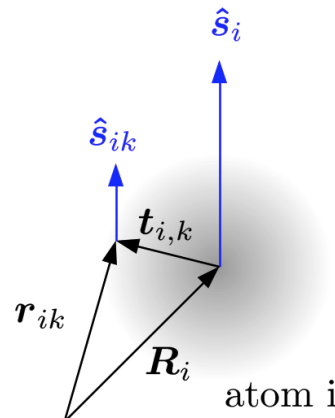
Example: Fe, low T, saturated

$$\mu = 2.2$$

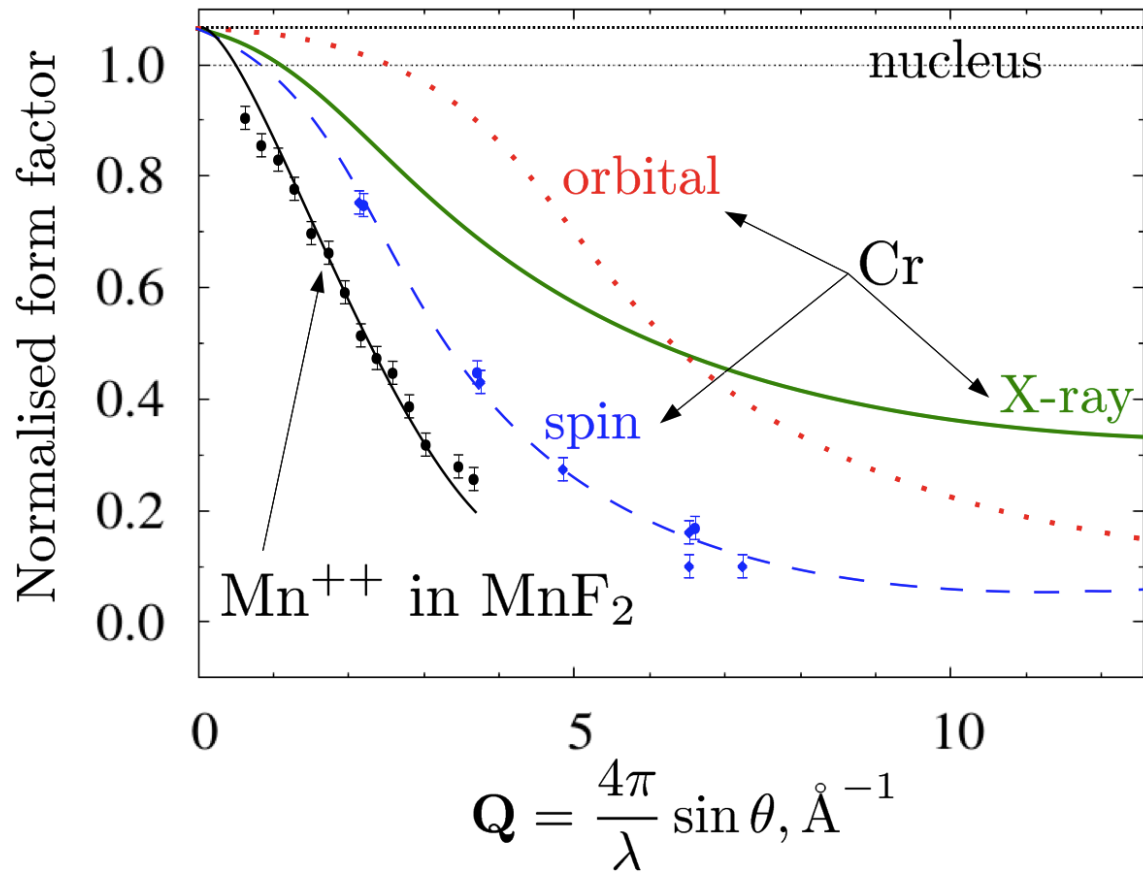
$$f(Q) = 0.59 \text{ @ } \sim 3.1 \text{\AA}^{-1}$$

$$p = 0.35 \times 10^{-12} \text{cm}$$

(analog to x-ray form factor
related to electron density)



Spin density form-factors of Cr and Mn



Neutron properties are suitable to study structure and dynamics of atoms and magnetic moments

Why polarized neutron scattering?

Neutron Diffraction by Paramagnetic and Antiferromagnetic Substances

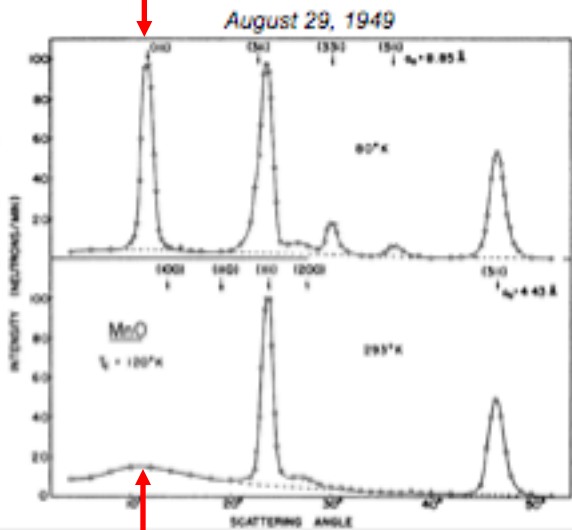
C. G. SHULL, W. A. STRAUER, AND E. O. WOLLAN
Oak Ridge National Laboratory, Oak Ridge, Tennessee
(Received March 2, 1951)



Detection of Antiferromagnetism by Neutron Diffraction 1949 *Shull et al. Nobel prize 1994*

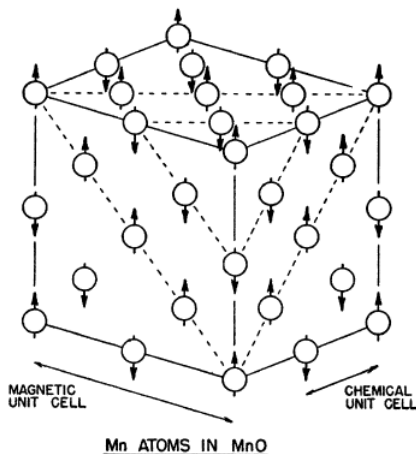
“for the development of neutron diffraction technique”

(111) alternating layers



paramagnetic spin fluctuations

MnO



spins

|| to [100]

Shull et al. PR 1951

in (111) planes

Shaked et al. PRB 1988

|| to $[1\bar{1}\bar{2}]$

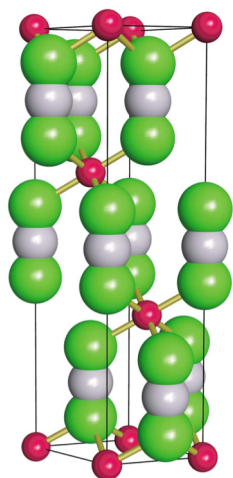
Goodwin et al. PRL 2006

MnO \Rightarrow MnNCN

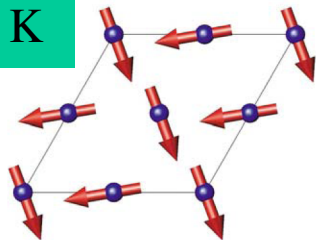
Replacing O^{-2} by NCN^{-2}

M. KROTT *et al* PHYSICAL REVIEW B **80**, 024117 2009

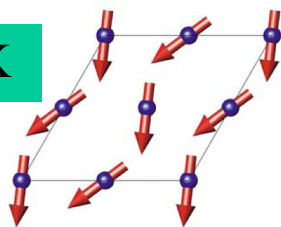
MnNCN



T = 4.5 K

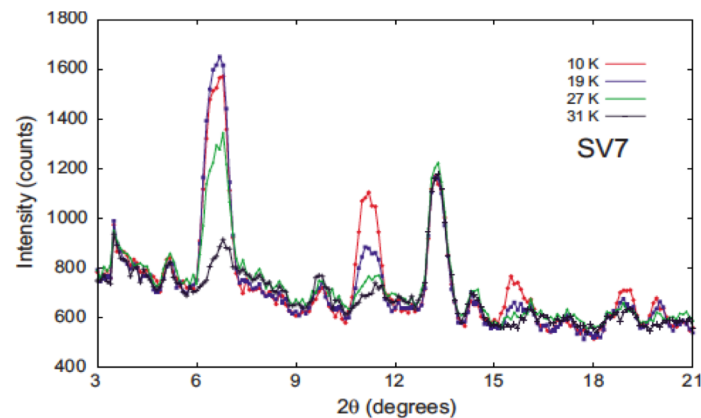


T = 20 K

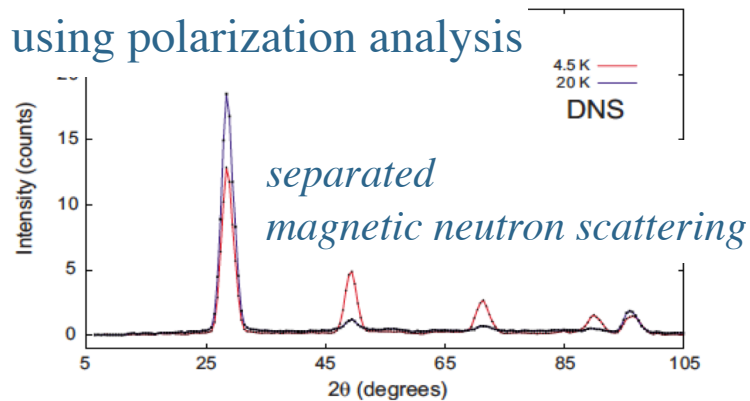


very clean results

conventional neutron diffraction



... using polarization analysis



Outline

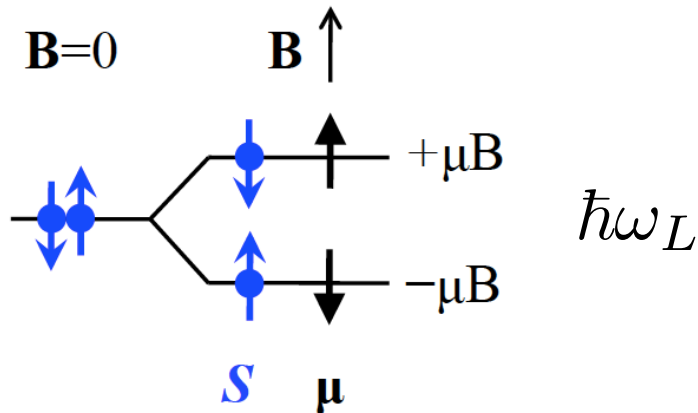
- **Neutron spins in magnetic fields**
 - toolbox of experimental devices => instruments
- **Scattering and Polarization**
 - spin-dependent nuclear interaction
 - magnetic interaction
- **Blume-Maleyev Equations**
 - examples
- **outlook for ESS**

$$\gamma = 2\gamma_n\mu_N/\hbar = -1.83 \cdot 10^8 \text{ s}^{-1}\text{T}^{-1}$$

$$\gamma/2\pi = -2916 \text{ Hz/Oe.}$$

Neutron spins in magnetic fields

Zeeman splitting



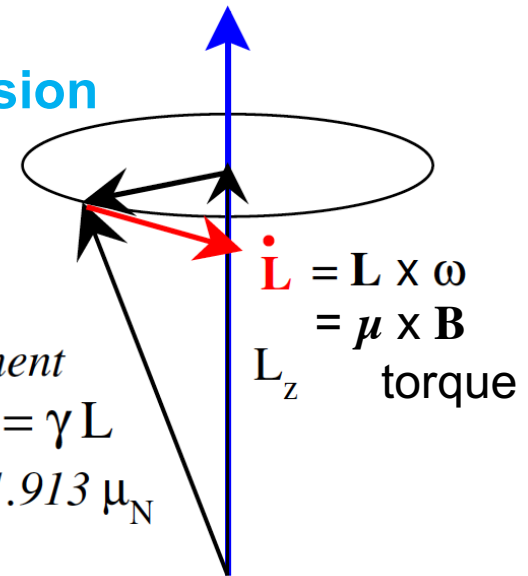
Larmor precession

$$\omega_L = -\gamma\mathbf{B}$$

*neutron
magnetic moment*

$$\mu = \gamma\mathbf{L}$$

$$= -1.913 \mu_N$$



Bloch equation of motion

$$\dot{\boldsymbol{\mu}} = \gamma \boldsymbol{\mu} \times \mathbf{B}$$

Neutron beam polarization

expectation value
average of spins:

$$\mathbf{P} = 2\langle \mathbf{S} \rangle \quad -1 < |\mathbf{P}| < 1$$

for a specific quantization axis

$$P = \frac{n_{\uparrow} - n_{\downarrow}}{n_{\uparrow} + n_{\downarrow}} \quad 0 < P < 1$$

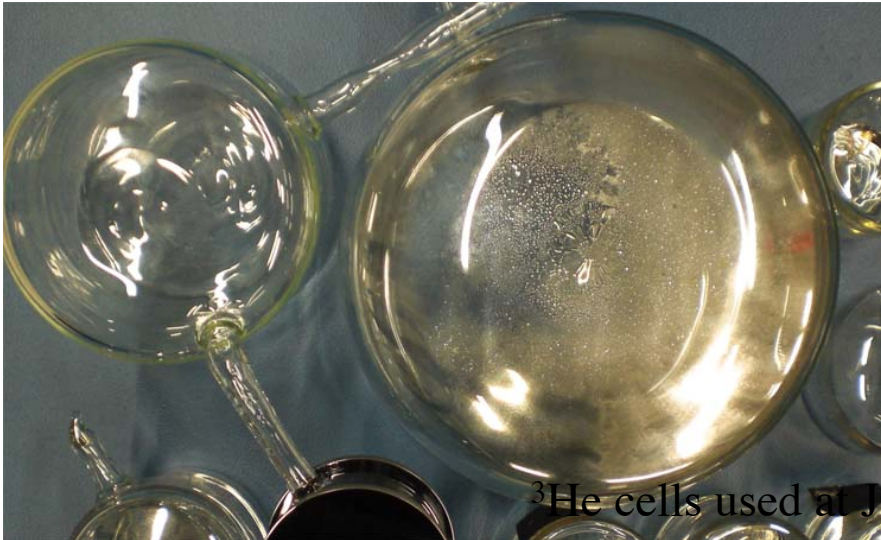
Neutron beam polarization

Absorption and transmission

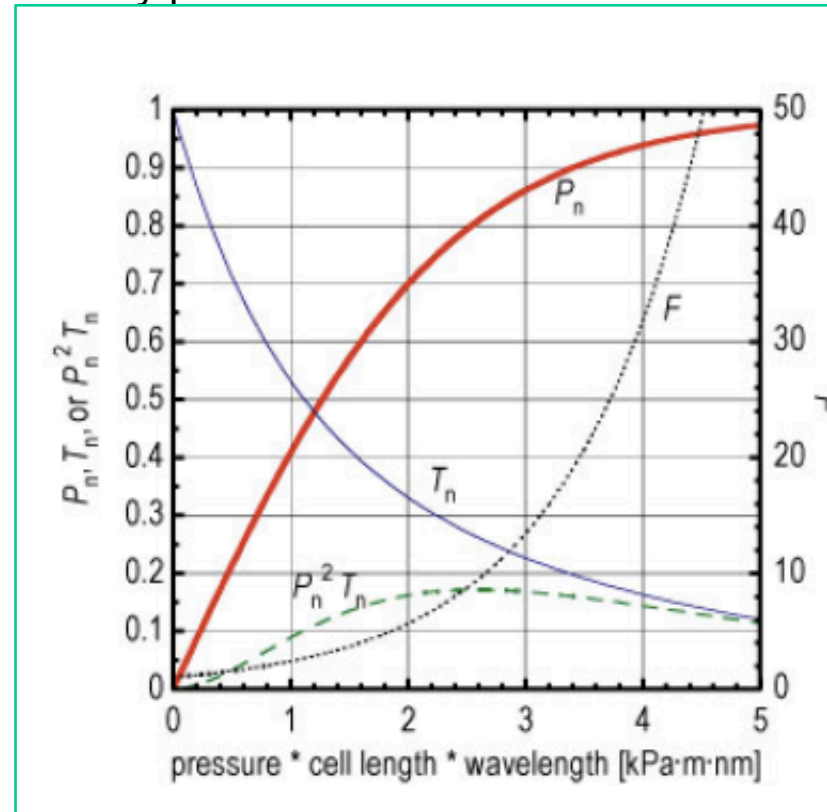
Polarized He-3 filter $\sigma_{abs}(n_{\uparrow}, {}^3\text{He}_{\uparrow}) = 0$

$$\sigma_{abs}(n_{\uparrow}, {}^3\text{He}_{\downarrow}) = 5333b$$

- SEOP Spin-exchange-optical pumping
- Laser polarizes Rb
- exchange with K then ${}^3\text{He}$ -spin
- very homogeneous field



Tunable efficiency
by pressure and volume



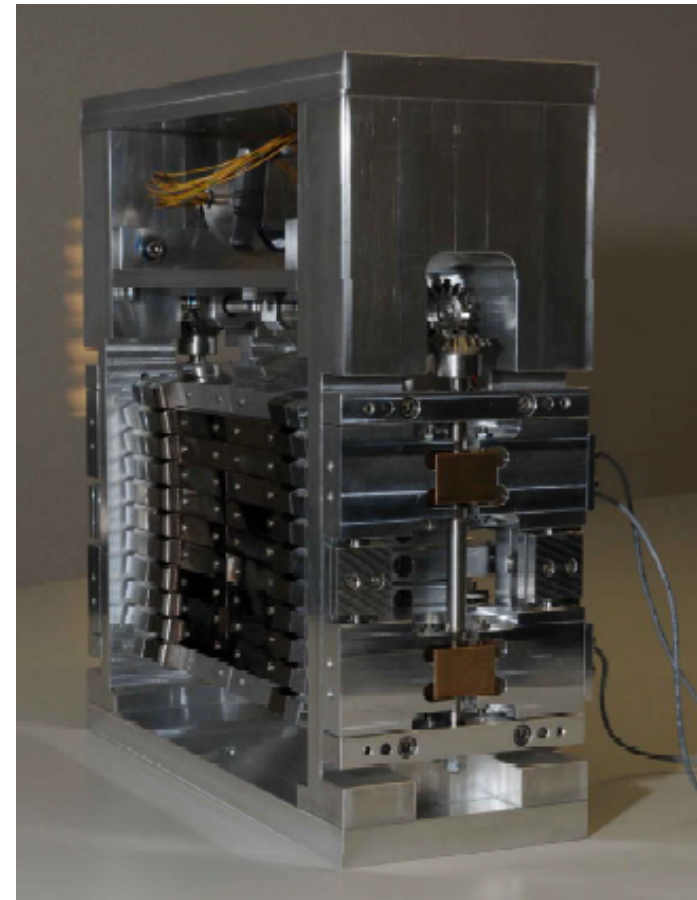
Neutron beam polarization

Scattering

constructive interference of nuclear and magnetic scattering

$$\sigma_{\pm} \propto (b \pm p)^2$$

-
- **Magnetic Bragg scattering**
e.g. Heusler crystals, Cu_2MnAl (111), $P = 0.95$
single ferro domain needed, low reflectivity



Neutron beam polarization

Scattering

constructive interference of nuclear and magnetic scattering

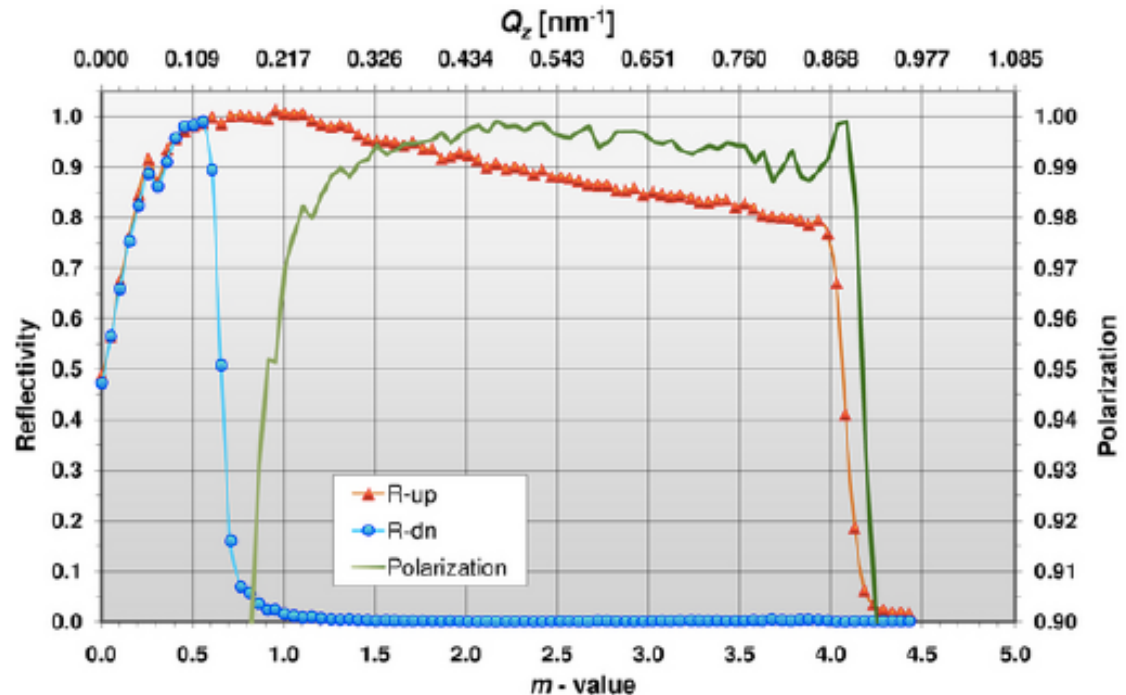
$$\sigma_{\pm} \propto (b \pm p)^2$$

- Total reflection by of magnetic “super-mirrors” (Mezei, Schärpf)

$$\Theta_c^{\pm} = \lambda \sqrt{n(b \pm p)/\pi}$$

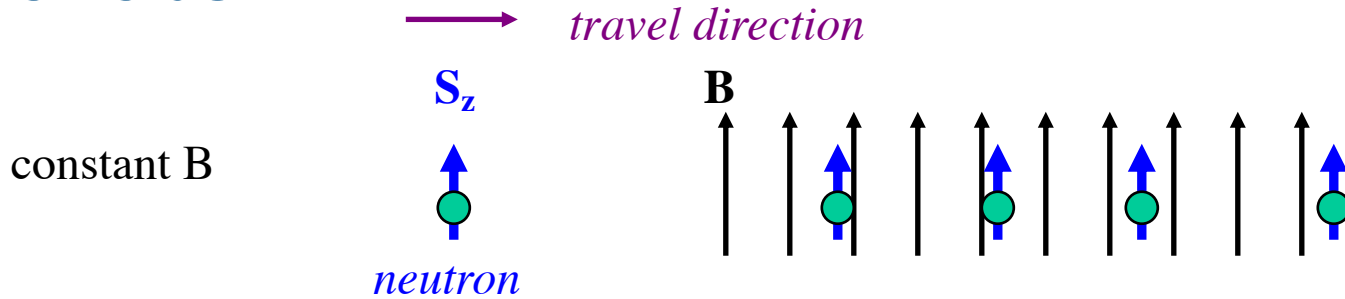
Surface of
FeSi multilayers

*much better polarization
at the interface of
Si : FeSi multilayers*



Source: Swiss Neutronics

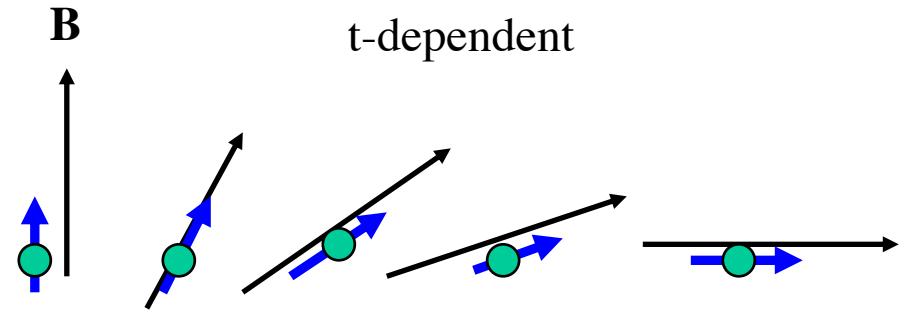
Guide fields



Asymptotic behaviour

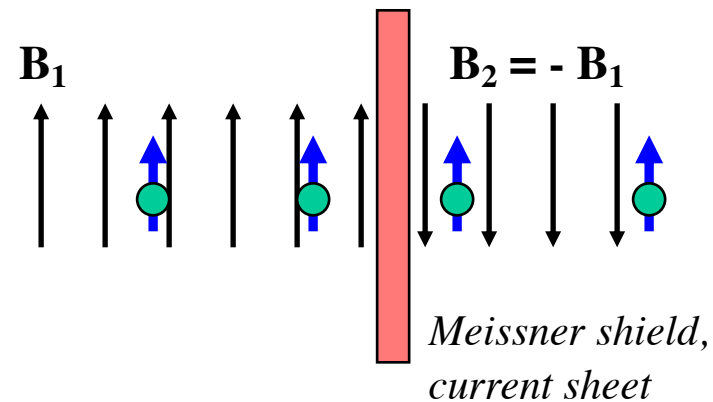
adiabatic field change

slowly varying “strong” $B(t)$
fast precession around B



sudden change

no change of S_z but change wrt B



General behaviour

Solve Bloch equation of motion $\dot{\mu} = \gamma \mu \times B$

Guide fields - adiabatic field change

nutators, xyz-coils

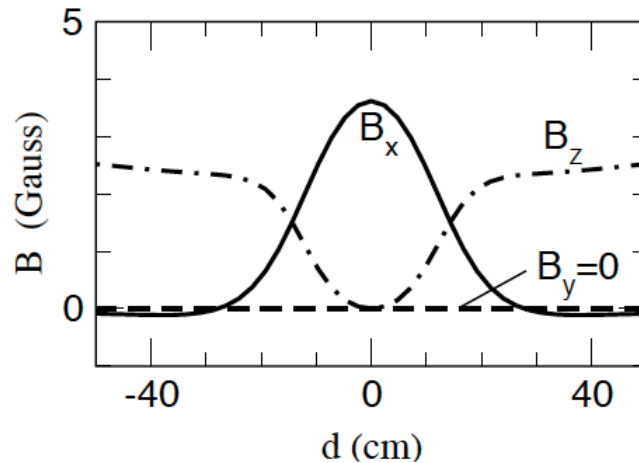
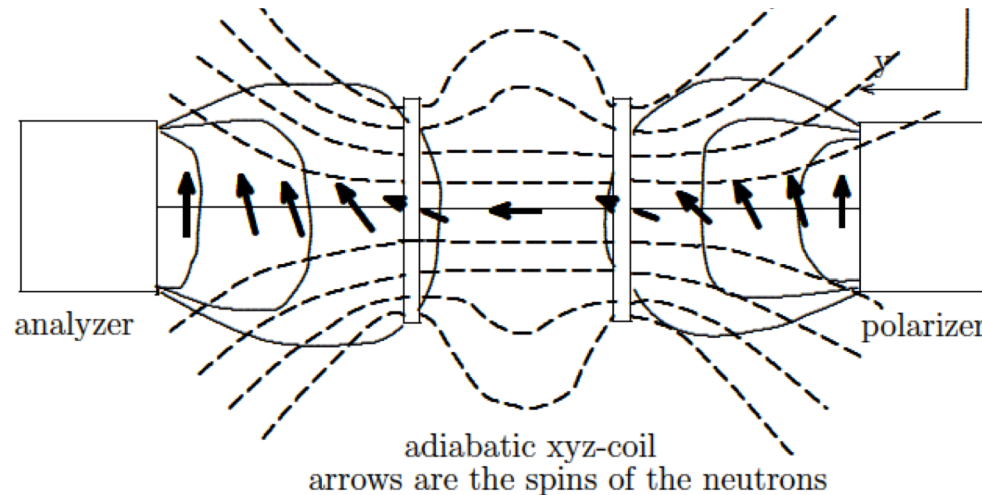
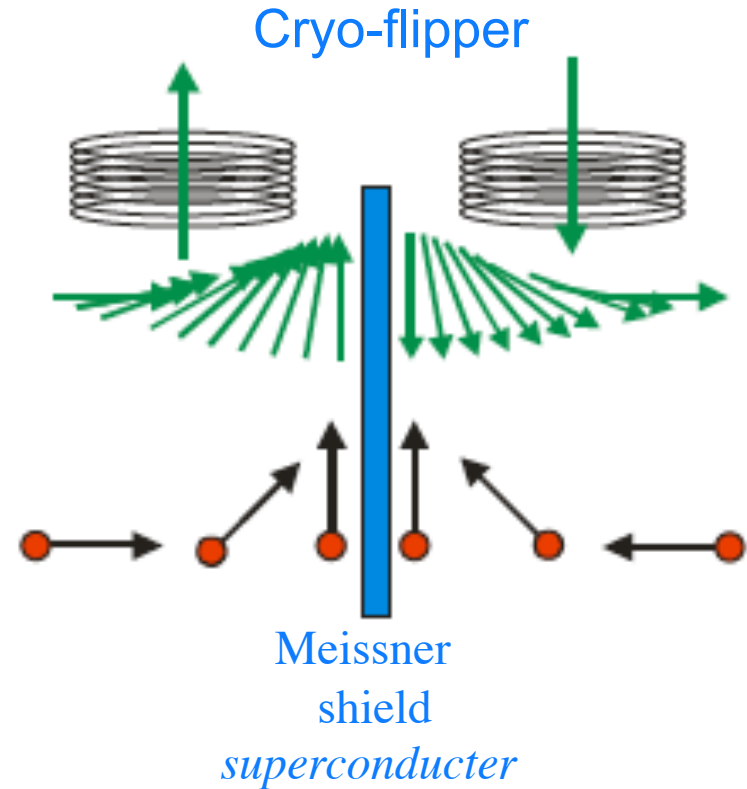
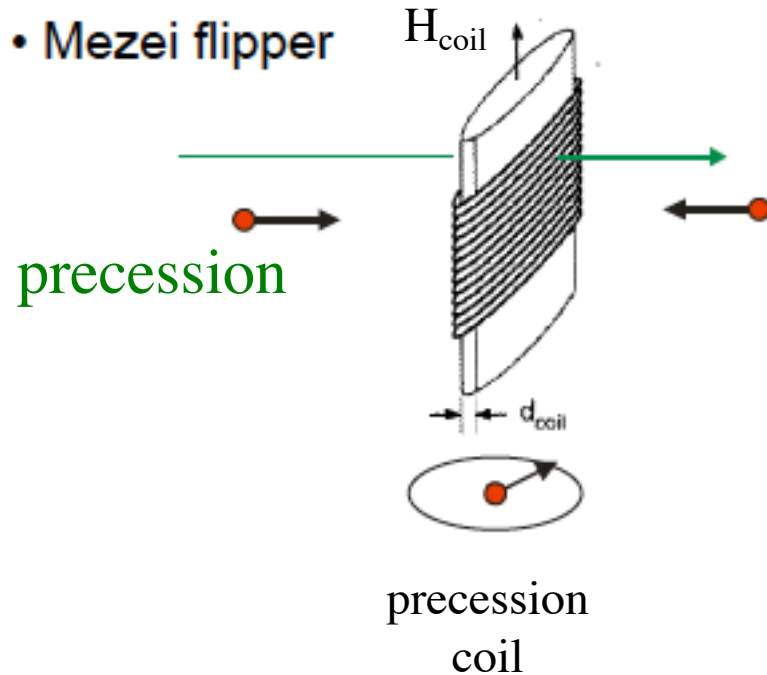


Fig. 6: (left) Magnetic field setting in a xyz-coil system for an adiabatic nutation of the polarization of cold neutrons in horizontal x -direction at the sample turning to a vertical (guide) field B_z at further distance from the sample. (right) A photo of the xyz-coil system in the DNS instrument at the FRM-2.

Flipper

Objective: change neutron polarization with respect to the applied field

Adiabatic and non-adiabatic changes of $P \parallel B$



π - flipper $\omega \cdot t = -\gamma B \cdot d/v = \pi$

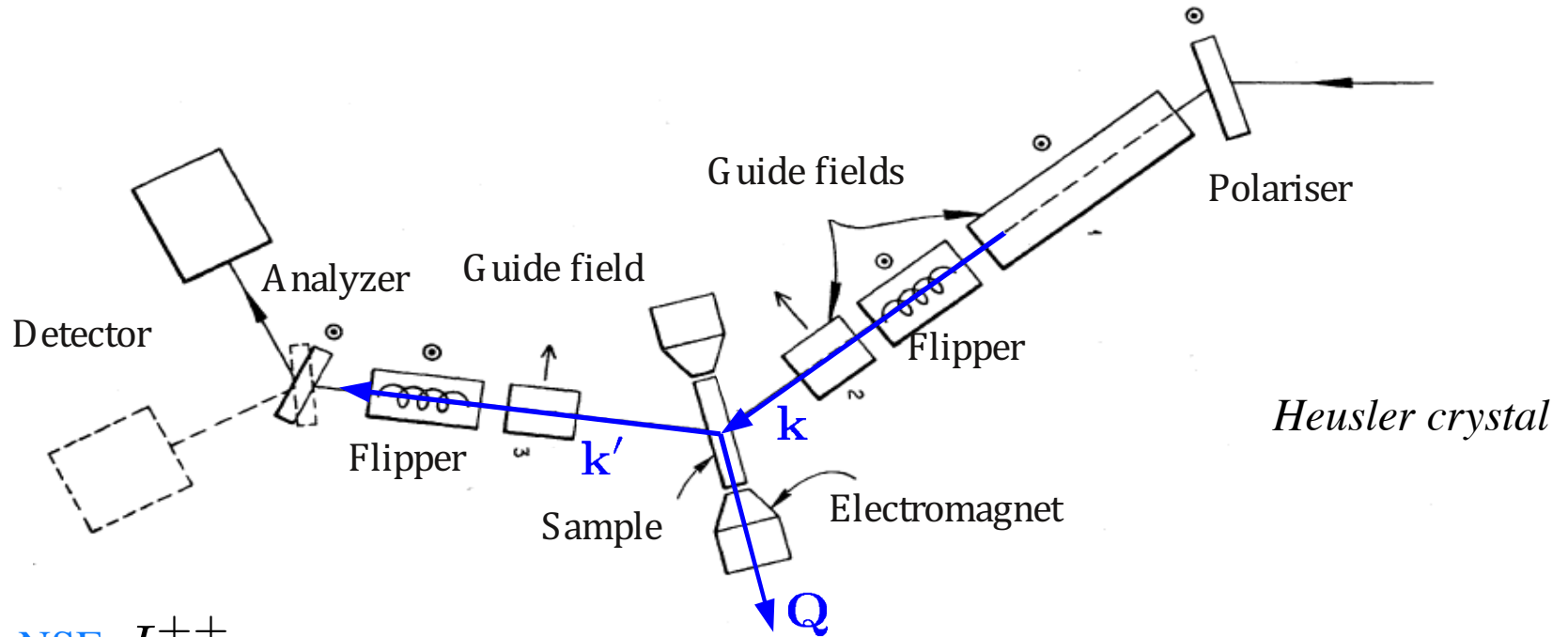
with $\gamma/2\pi = -2916 \text{ Hz/Oe}$ and $mv = h/\lambda$

$$B = \frac{\pi}{d} (3956 \text{ m/s} \cdot \text{\AA}/\lambda) / (2916 \cdot 2\pi \text{ Hz/Oe}) = \frac{67.83}{d\lambda} \text{ cm \AA Oe}$$

Polarization Analysis of Thermal-Neutron Scattering*

R. M. MOON, T. RISTE,[†] AND W. C. KOEHLER

Solid State Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830



NSF I^{++}

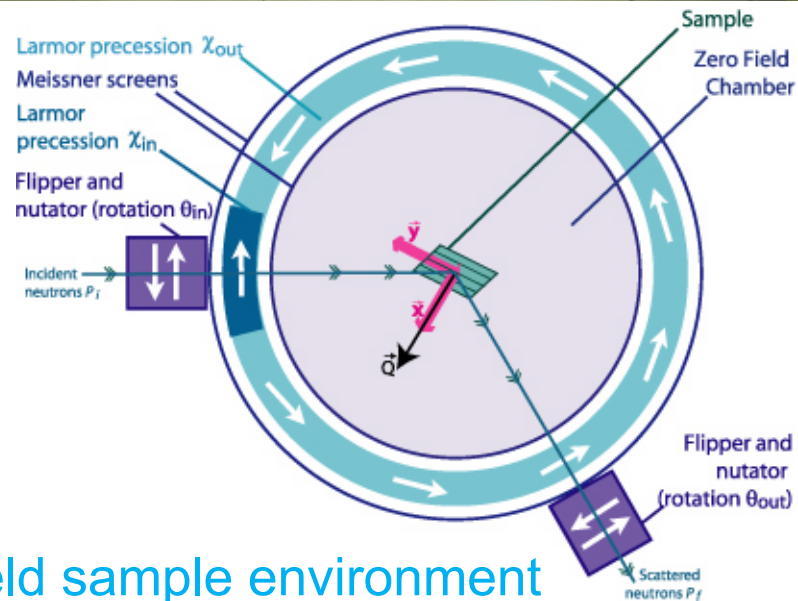
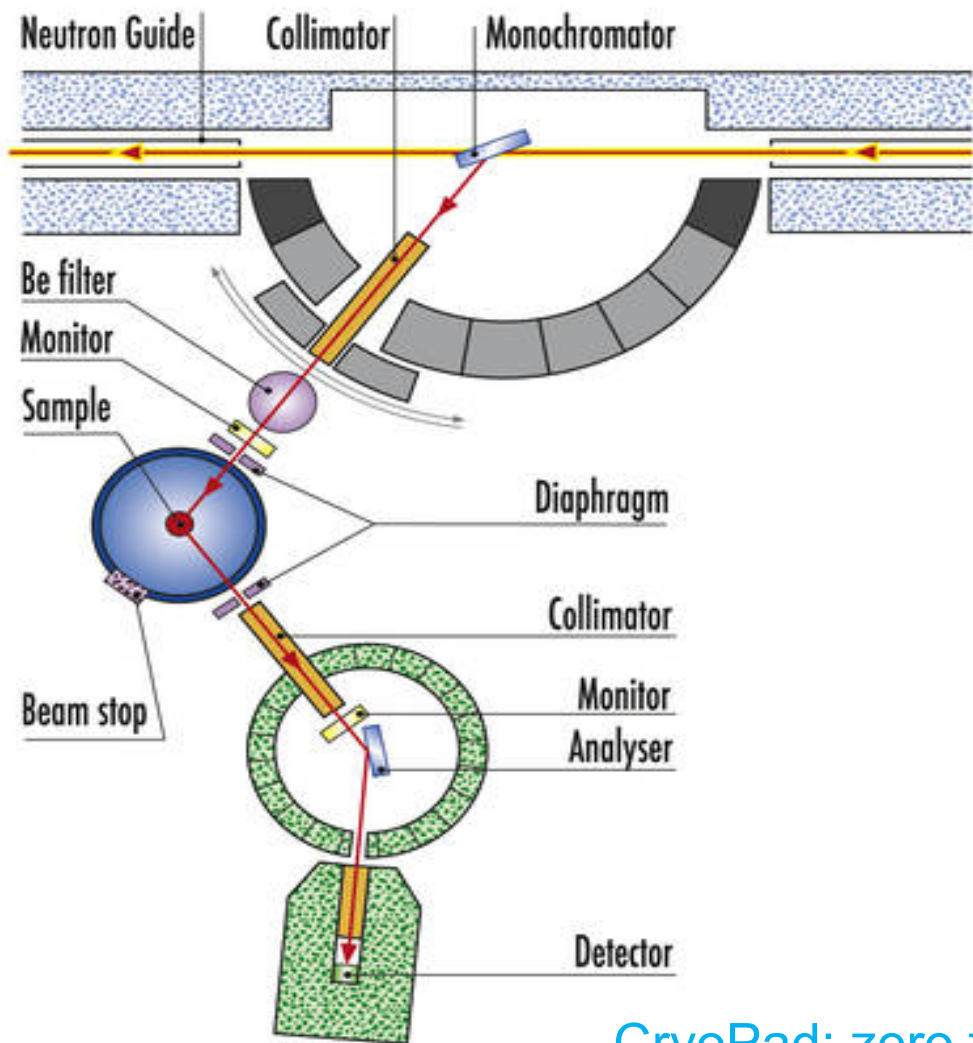
SF I^{+-}

Triple axis instrument

with longitudinal polarization analysis $P \parallel H$

Triple axis instrument with *spherical* polarization analysis

IN12 @ ILL



CryoPad: zero field sample environment

Outline

- **Neutron spins in magnetic fields**
 - toolbox of experimental devices => instruments
- **Scattering and Polarization**
 - spin-dependent nuclear interaction
 - magnetic interaction
- **Blume-Maleyev Equations**
 - examples
- **outlook for ESS**

Coherent nuclear scattering

Differential scattering cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{m_n}{2\pi\hbar}\right)^2 |\langle \mathbf{k}' \mathbf{S}' | V | \mathbf{k} \mathbf{S} \rangle|^2$$

$$V(\mathbf{r}) = \frac{2\pi\hbar^2}{m} b \delta(\mathbf{r} - \mathbf{R})$$

$$\langle \mathbf{k}' | V | \mathbf{k} \rangle = \frac{2\pi\hbar^2}{m} \sum_l b_l e^{i\mathbf{Q}\mathbf{R}_l}$$

$$= b(\mathbf{Q})$$

including initial and final spin states

1st Born approximation

Point like nucleus

Conservation of momentum and plane wave scattering

assuming the nuclei have no spin

Scattering amplitude – transition matrix element

$$A(\mathbf{Q}) = \langle S'_Z | b(\mathbf{Q}) | S_Z \rangle = b(\mathbf{Q}) \langle S'_Z | S_Z \rangle$$

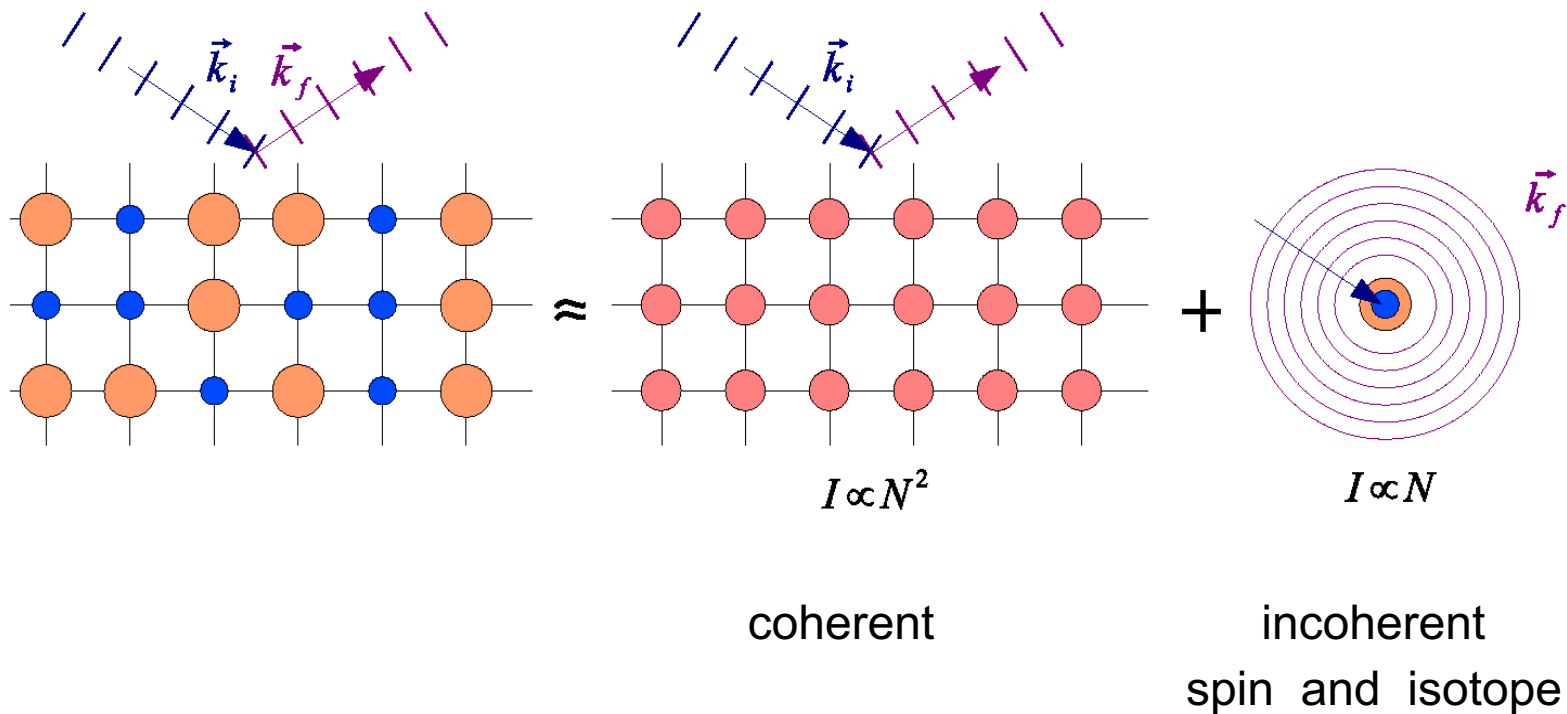
$$= b(\mathbf{Q}) \quad \text{no spin-flip}$$

$$= 0 \quad \text{spin-flip}$$

$$\frac{d\sigma}{d\Omega} = \bar{b}^2 \sum_{l,l'} e^{i\mathbf{Q}(\mathbf{R}_l - \mathbf{R}_{l'})}$$

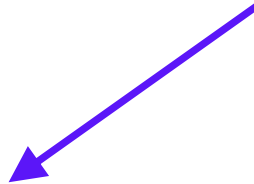
Coherent & incoherent scattering

$$\frac{d\sigma}{d\Omega} = \bar{b}^2 \sum_{ll'} e^{i\mathbf{Q}(\mathbf{R}_l - \mathbf{R}_{l'})} + N (\bar{b}^2 - \bar{b}^2)$$



Spin dependent nuclear scattering amplitude

$$A(\mathbf{Q}) = \langle \mathbf{k}' \mathbf{S}' | A + B \hat{\sigma} \cdot \hat{\mathbf{I}} | \mathbf{k} \mathbf{S} \rangle$$



Spin operator $\hat{\sigma} = \{ \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z \}$

Pauli Matrices $\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

spin states, quantization axis z $|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\hat{\sigma}_x |+\rangle = |-\rangle \quad \hat{\sigma}_x |-\rangle = |+\rangle \quad 2/3 \text{ spinflip}$$

$$\hat{\sigma}_y |+\rangle = i|-\rangle \quad \hat{\sigma}_y |-\rangle = -i|+\rangle$$

$$\hat{\sigma}_z |+\rangle = |+\rangle \quad \hat{\sigma}_z |-\rangle = -|-\rangle \quad 1/3 \text{ non-spinflip}$$

Spin dependent nuclear scattering amplitude

$$A(\mathbf{Q}) = \langle \mathbf{k}' \mathbf{S}' | A + B \hat{\sigma} \cdot \hat{\mathbf{I}} | \mathbf{k} \mathbf{S} \rangle$$

$$I = 0 \quad A(\mathbf{Q}) = \langle S'_z | \bar{b} | S_z \rangle = \bar{b} \langle S'_z | S_z \rangle$$

$$\langle + | + \rangle = \langle - | - \rangle = 1$$

$$\langle + | - \rangle = \langle - | + \rangle = 0$$

**No spin flip in absence
of a nuclear spin**

$$I \neq 0 \quad A(\mathbf{Q})^{\text{NSF}} = A + BI_z \quad \text{for the } ++ \text{ and } -- \text{ case}$$
$$A(\mathbf{Q})^{\text{SF}} = B(I_x + iI_y) \quad \text{for the } +- \text{ and } -+ \text{ case}$$

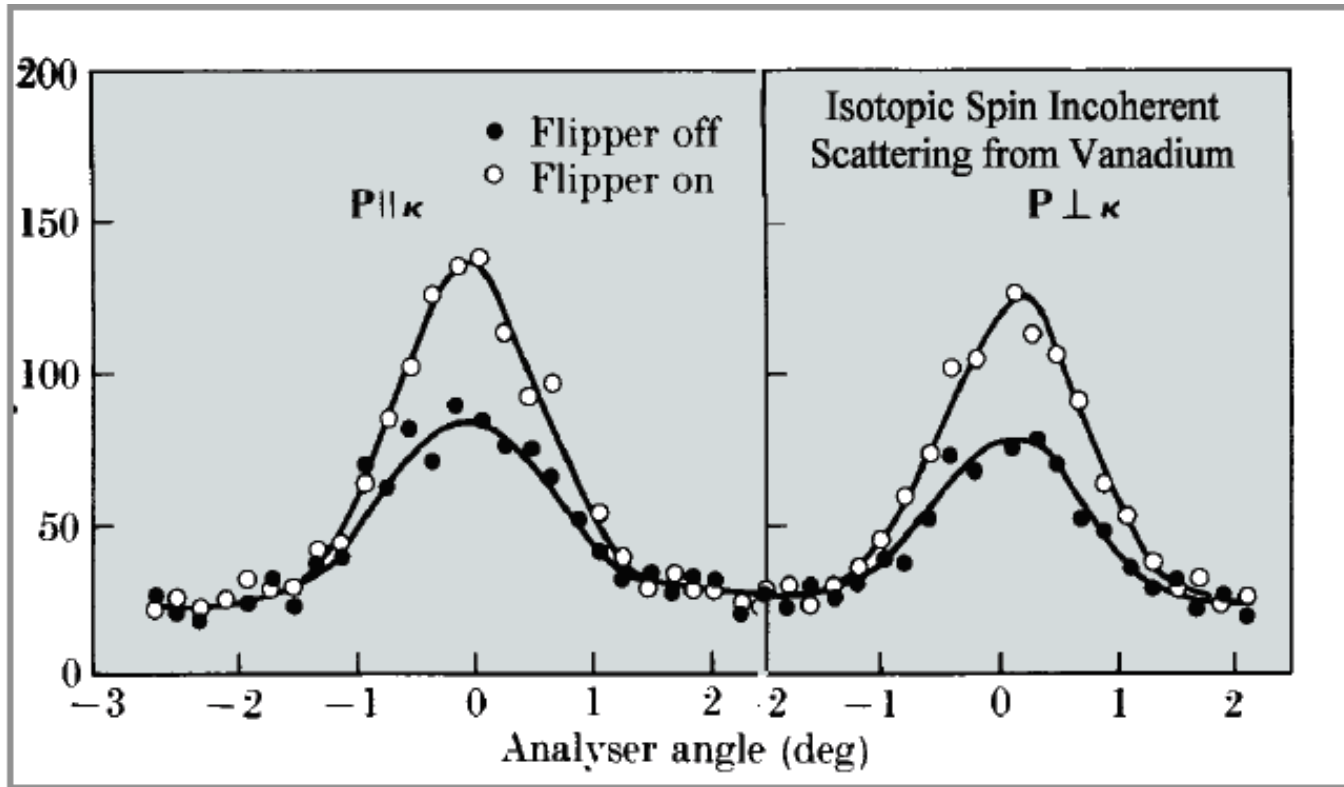
A perpendicular nuclear spin flips the neutron spin!

A parallel nuclear spins flip does not

2/3 of spin-incoherent scattering is spin-flip

for disordered nuclear spins, independent of the direction of \mathbf{P}

Moon, Riste and Koehler (1969)



2/3 of spin-incoherent scattering is spin-flip independent of the direction of P

Polarization analysis: Spin-flip and non-spin-flip scattering

Separation of spin-incoherent and coherent nuclear scattering

Applications to hydrogenous materials, soft matter, etc.

$$\frac{d\sigma^N}{d\Omega_{Q,coh}} + \frac{d\sigma^N}{d\Omega_{isotop-inc}} \stackrel{\text{small}}{=} \frac{d\sigma^{NSF}}{d\Omega} - \frac{1}{2} \frac{d\sigma^{SF}}{d\Omega}$$

$$\frac{d\sigma}{d\Omega_{spin-inc}} = \frac{3}{2} \frac{d\sigma^{SF}}{d\Omega}$$

$$\sigma_{coh}^H = 1.75b \quad \sigma_{inc}^H = 80.26b$$

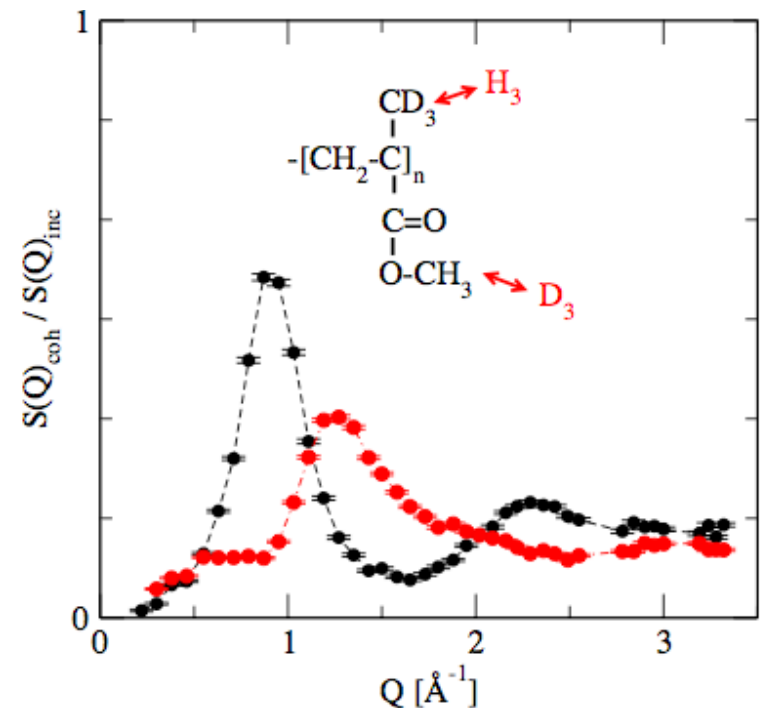
$$b_{coh}^H = -3.74 \text{ fm} \quad b_{coh}^D = +6.67 \text{ fm}$$

* Separating huge incoherent background of H

* Intrinsic calibration

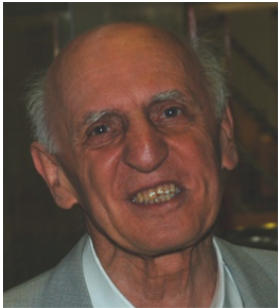
PMMA

DNS at FRM II



from intensities to partial pair-correlation functions
to compare with MD and MC simulations

Liquid sodium at 840 K
 (homepage Otto Schärpf)



Neutron scattering lengths and cross sections							
Isotope	conc	Coh b	Inc b	Coh xs	Inc xs	Scatt xs	Abs xs
Na	100	3.63	3.59	1.66	1.62	3.28	0.53

www.ncnr.nist.gov

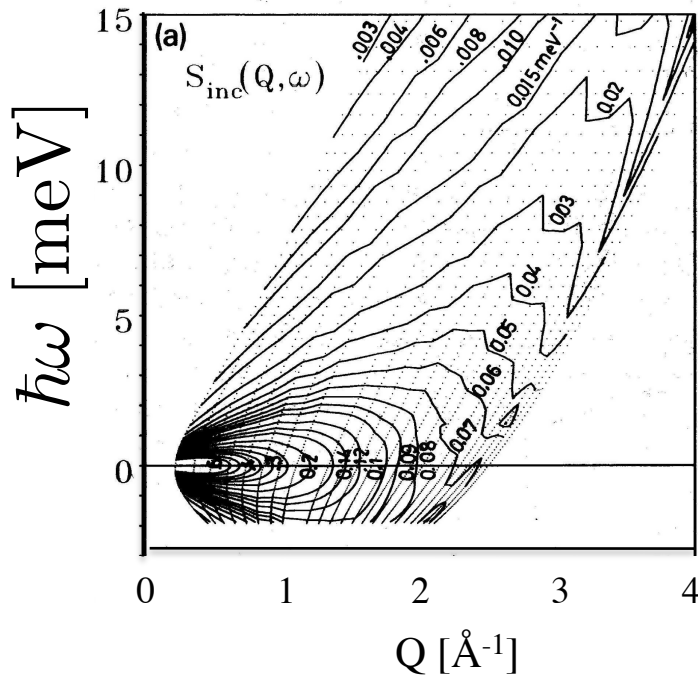
a good motivation to think about how to separate scattering

spin-incoherent

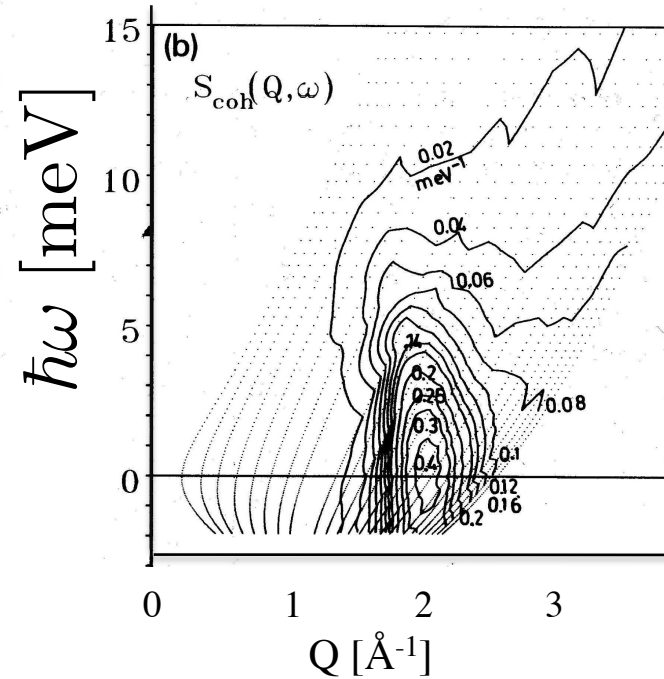
$$\sigma_{inco} = 1.62b$$

coherent

$$\sigma_{coh} = 1.66b$$



FT (self correlation)
 single particle diffusion



FT (pair correlation)
 collective behavior
 precursors of Bragg scattering

Magnetic scattering cross section

initial and final spin states

$$\begin{aligned}
 A(\mathbf{Q}) &= \langle S'_z | -\frac{\gamma_n r_0}{2\mu_B} \hat{\boldsymbol{\sigma}} \cdot \mathbf{M}_\perp(\mathbf{Q}) | S_z \rangle \\
 &= -\frac{\gamma_n r_0}{2\mu_B} \sum_\alpha \langle S'_z | \hat{\boldsymbol{\sigma}}_\alpha | S_z \rangle \mathbf{M}_{\perp\alpha}(\mathbf{Q}) \quad \alpha = x, y, \text{ or } z
 \end{aligned}$$

$$\hat{\boldsymbol{\sigma}}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{\boldsymbol{\sigma}}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{\boldsymbol{\sigma}}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Choosing z as quantization axis

$$A(\mathbf{Q}) = -\frac{\gamma_n r_0}{2\mu_B} \cdot \begin{cases} \mathbf{M}_{\perp\mathbf{Q},z} & \text{for the } ++ \text{ NSF case} \\ -\mathbf{M}_{\perp\mathbf{Q},z} & \text{for the } -- \text{ NSF case} \\ \mathbf{M}_{\perp\mathbf{Q},x} - i\mathbf{M}_{\perp\mathbf{Q},y} & \text{for the } +- \text{ SF case} \\ \mathbf{M}_{\perp\mathbf{Q},x} + i\mathbf{M}_{\perp\mathbf{Q},y} & \text{for the } -+ \text{ SF case} \end{cases}$$

coordinate system say $x \parallel \mathbf{Q}$

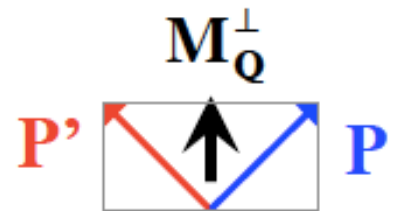
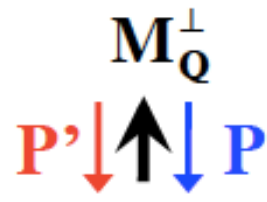
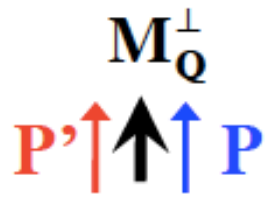
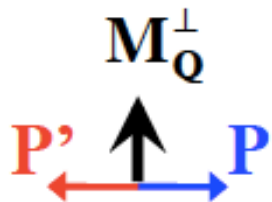
we have seen this before:

A perpendicular component flips the neutron spin!
A parallel component does not

direction of \mathbf{P} , \mathbf{M} , \mathbf{Q} matters!

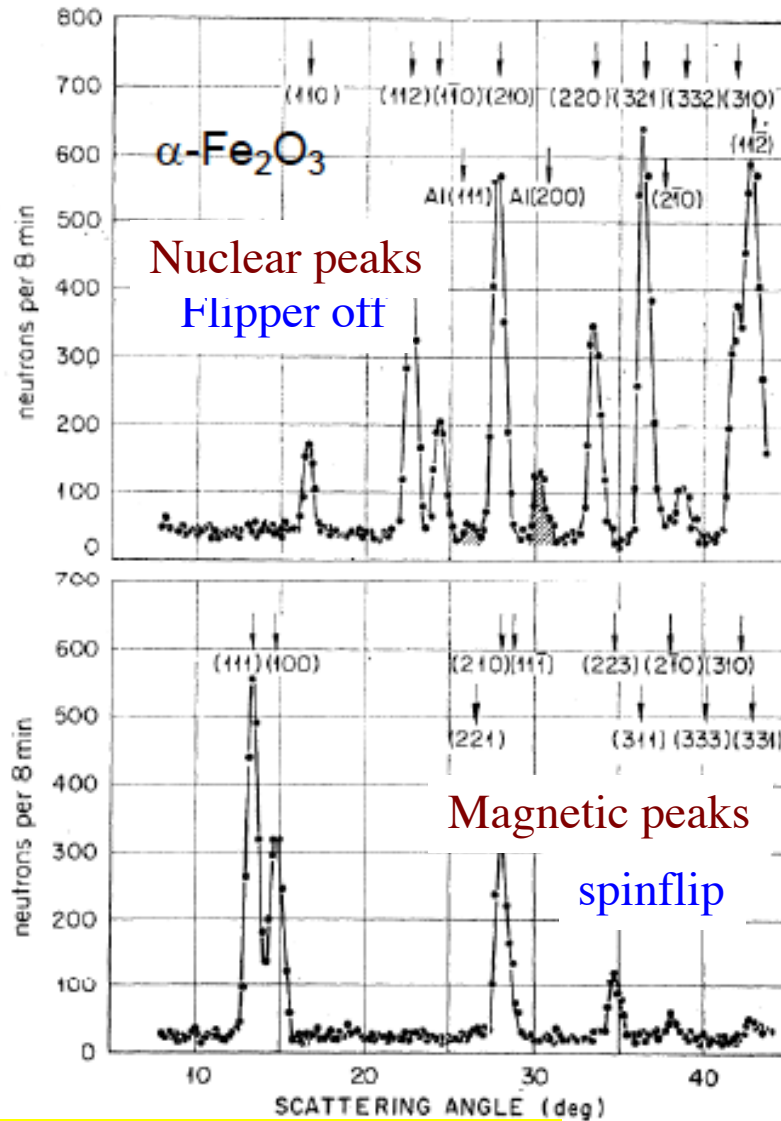
$$\langle + | \hat{\sigma} \cdot \hat{M}_Q^\perp | + \rangle = M_{z,Q}^\perp$$

$$\langle - | \hat{\sigma} \cdot \hat{M}_Q^\perp | + \rangle = iM_{y,Q}^\perp$$



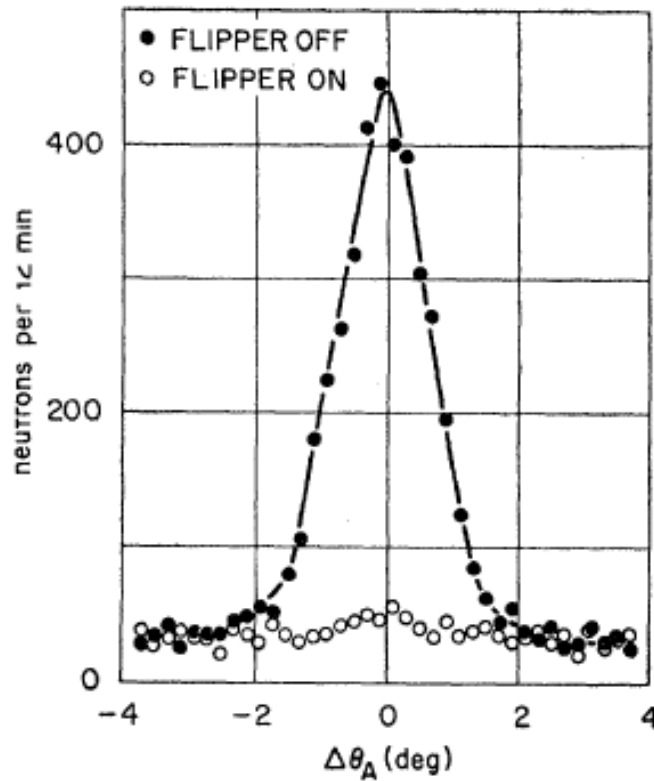
Separating nuclear and magnetic scattering

- Antiferromagnet: Bragg scattering



Moon, Riste and Koehler (1969)

Ni – Ferromagnet



Isotopic incoherent scattering

$$\mathbf{P} \parallel \mathbf{Q}$$

no magnetic scattering
for $\mathbf{M} \parallel \mathbf{Q}$

Typically, FM need saturating fields

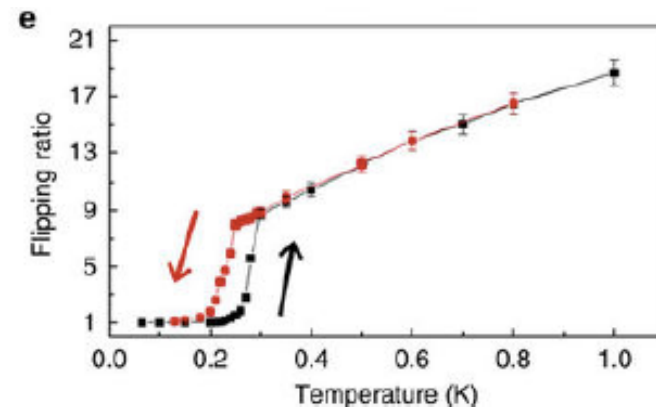
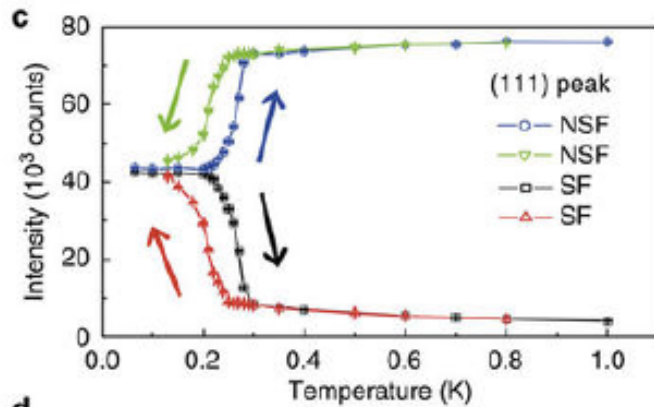
Note: thermocouple alloy $\text{Ni}_{89}\text{Cr}_{11}$ is non-magnetic

Depolarisation of the neutron spins are observed ...

Higgs transition from a magnetic Coulomb liquid to a ferromagnet in $\text{Yb}_2\text{Ti}_2\text{O}_7$

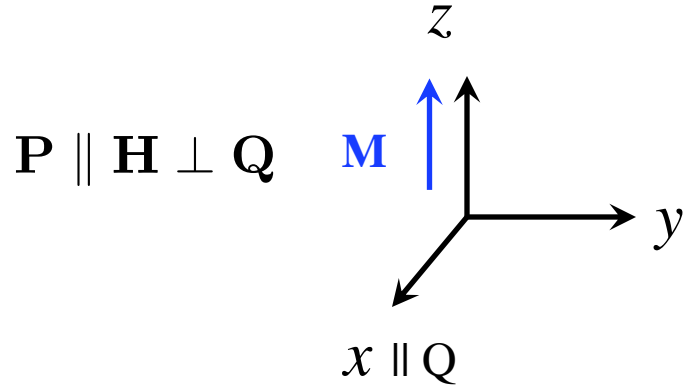
LJ Chang et al, Nature Communications 2012

depolarization of the neutron spins are observed with thermal hysteresis, indicating a first-order ferromagnetic transition. Our results are explained on the basis of a quantum spin-ice model, whose high-temperature phase is effectively described as a magnetic Coulomb liquid, whereas the ground state shows a nearly collinear ferromagnetism with gapped spin excitations.

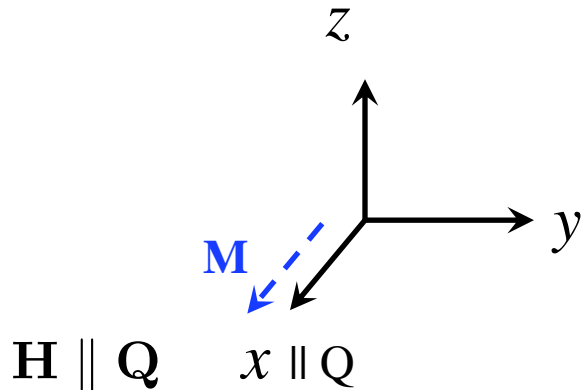


Ferromagnets: Separation by the \parallel - \perp method, case of strong field H

FM



nsf	$ M_z ^2 + N ^2 \pm 2M_z N$	$+\frac{1}{3}\sigma_{si}$
sf		$+\frac{2}{3}\sigma_{si}$



nsf	$ N ^2$	$+\frac{1}{3}\sigma_{si}$
sf		$+\frac{2}{3}\sigma_{si}$

Note: For ferromagnets, the magnetic scattering can be separated by the difference due to field variation even without polarization analysis.
The interference term $M_z N$ requires P, however, no polarization analysis

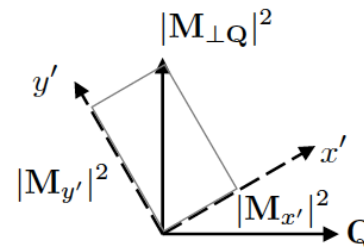
Powder diffraction with “XYZ” polarization analysis

⇒ Separation of
magnetic and nuclear scattering
and spin-incoherent background

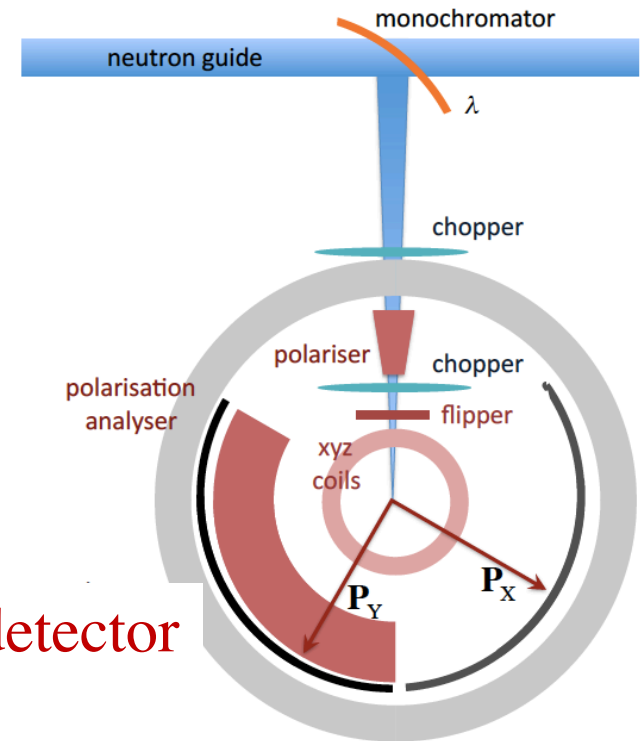
DNS at MLZ, Munich
(~ D7 at ILL, Grenoble)



O. Schärpf and H. Capellmann,
Phys. Status Solidi A 135, 359 (1993).



multi-detector



$$\frac{d\sigma}{d\Omega_{magn}} = 2 \left(\frac{d\sigma^{SF}}{d\Omega_x} + \frac{d\sigma^{SF}}{d\Omega_y} - 2 \frac{d\sigma^{SF}}{d\Omega_z} \right) = -2 \left(\frac{d\sigma^{NSF}}{d\Omega_x} + \frac{d\sigma^{NSF}}{d\Omega_y} - 2 \frac{d\sigma^{NSF}}{d\Omega_z} \right)$$

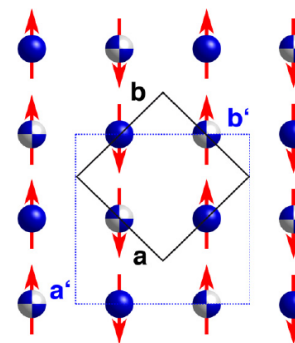
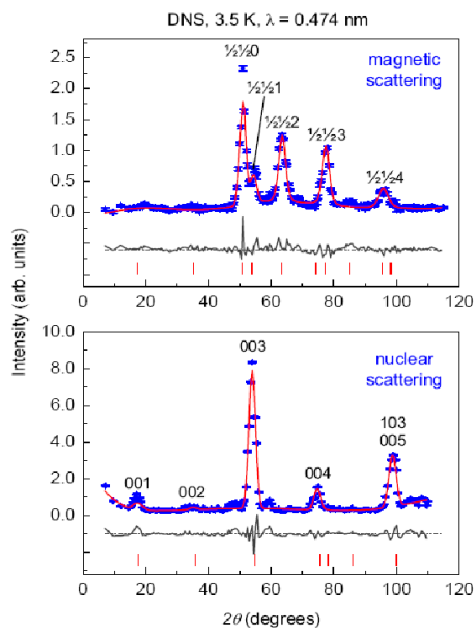
Powder diffraction with “XYZ” polarization analysis

⇒ Separation of magnetic and nuclear scattering and spin-incoherent background

DNS at MLZ, Munich
(~ D7 at ILL, Grenoble)



Non-stoichiometry and the magnetic structure of $\text{Sr}_2\text{CrO}_3\text{FeAs}$



M. Tegel et al, Europhysics Lett. 2010

FIG. 3: Magnetic and nuclear reflections of $\text{Sr}_2\text{CrO}_3\text{FeAs}$ (blue) and Rietveld fit (red) at 3.5 K measured at DNS [3].

new dedicated powder diffractometers
WOMBAT, ECHNIDA at ANSTO, Sidney

Diffuse magnetic scattering with “XYZ” polarization analysis

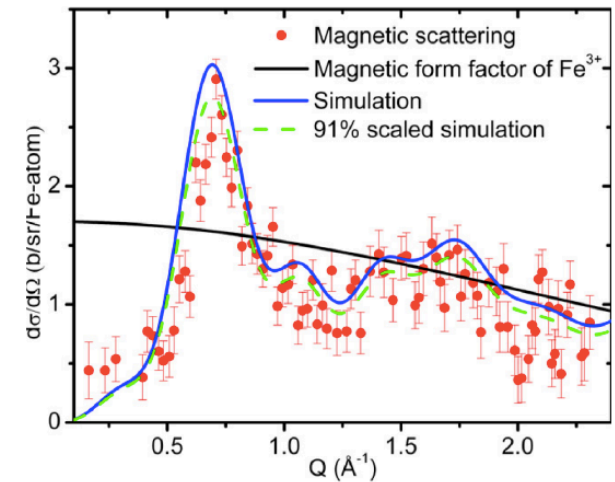
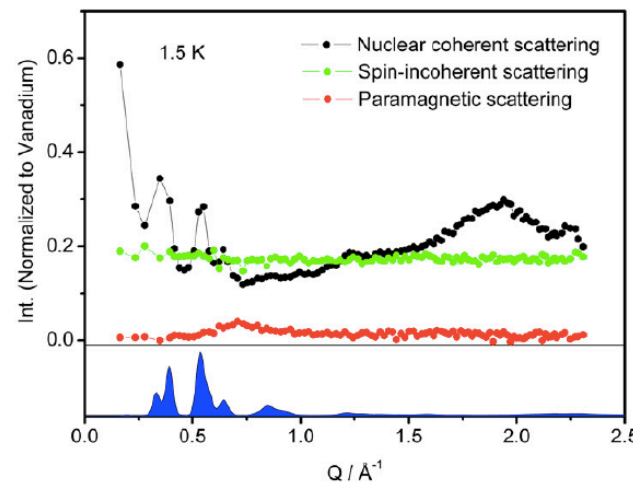
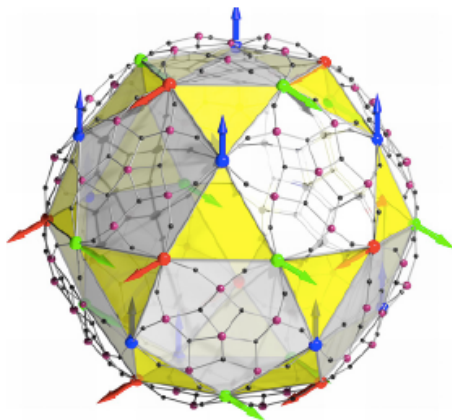
DNS at MLZ, Munich
(~ D7 at ILL, Grenoble)



⇒ Separation of
magnetic and nuclear scattering
and spin-incoherent background

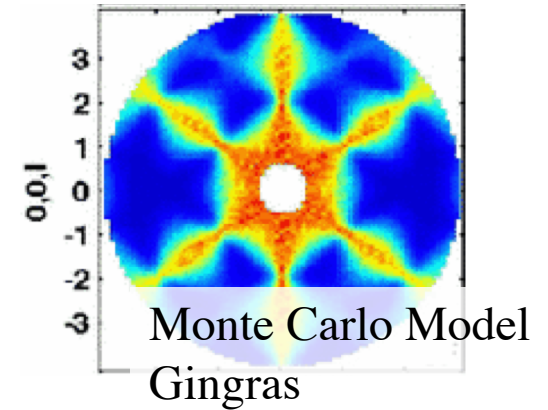
Magnetic ground state of the molecular magnet $[\text{Mo}_{72}\text{Fe}_{30}]$

Zhendong Fu et al, New Journal of Physics **12** (2010)

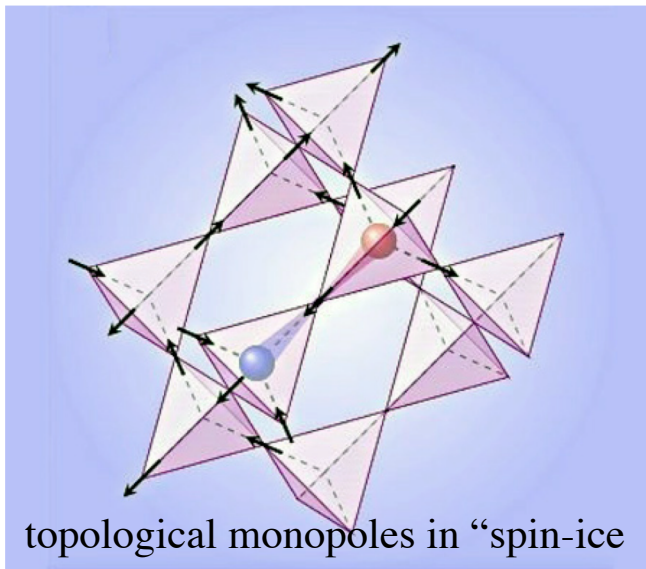


Single crystal diffuse magnetic scattering with “XYZ” polarization analysis

⇒ Separation of magnetic and nuclear scattering and spin-incoherent background

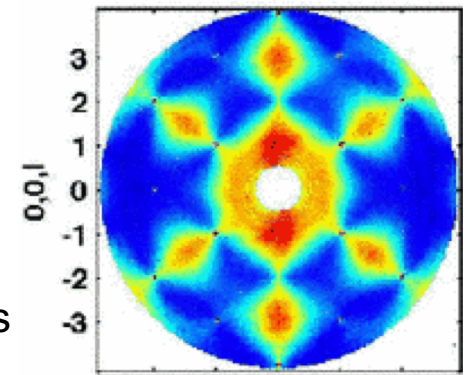


Magnetic Coulomb Phase in the Spin Ice $\text{Ho}_2\text{Ti}_2\text{O}_7$
T. Fennell et al. Science 2009

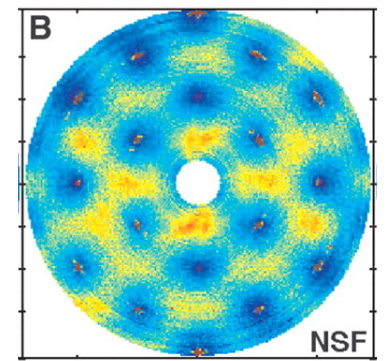


Poles apart. A pyramid with three ions pointing in (blue) acts as a north monopole; one with one ion pointing in (red) acts as a south monopole. By flipping other spins, the monopoles can be moved apart.

$\mathbf{P} \parallel z$
Spin-flip
Scattering
shows
pinch-points



NSF



$h, h, 0$

Outline

- **Neutron spins in magnetic fields**
 - experimental devices => instruments
- **Scattering and Polarization**
 - spin-dependent nuclear interaction
 - magnetic interaction
- **Blume-Maleyev Equations**
 - examples
- **outlook for ESS**

Blume – Maleyev (1963) general theory for polarized neutron scattering

... yields two expressions

for **scattering intensity**

$$\begin{aligned} \sigma_{\mathbf{Q}} = & \sigma_{\mathbf{Q},\text{coh}}^{\text{N}} + \sigma_{\mathbf{Q},\text{isotope-inc}}^{\text{N}} + \sigma_{\mathbf{Q},\text{spin-inc}}^{\text{N}} \\ & + |\mathbf{M}_{\mathbf{Q}}^{\perp}|^2 + \mathbf{P}(N_{-\mathbf{Q}}\mathbf{M}_{\mathbf{Q}}^{\perp} + \mathbf{M}_{-\mathbf{Q}}^{\perp}N_{\mathbf{Q}}) + i\mathbf{P}(\mathbf{M}_{-\mathbf{Q}}^{\perp} \times \mathbf{M}_{\mathbf{Q}}^{\perp}) \\ & \textit{magnetic} \quad \textit{magnetic-nuclear interference} \quad \textit{chirality} \end{aligned}$$

and **final polarized intensity**

$$\begin{aligned} \mathbf{P}'\sigma_{\mathbf{Q}} = & \mathbf{P}\sigma_{\mathbf{Q},\text{coh}}^{\text{N}} + \mathbf{P}\sigma_{\mathbf{Q},\text{isotop-inc}}^{\text{N}} - \frac{1}{3}\mathbf{P}\sigma_{\mathbf{Q},\text{spin-inc}}^{\text{N}} \\ & + \mathbf{M}_{\mathbf{Q}}^{\perp}(\mathbf{P}\mathbf{M}_{-\mathbf{Q}}^{\perp}) + \mathbf{M}_{-\mathbf{Q}}^{\perp}(\mathbf{P}\mathbf{M}_{\mathbf{Q}}^{\perp}) - \mathbf{P}\mathbf{M}_{\mathbf{Q}}^{\perp}\mathbf{M}_{-\mathbf{Q}}^{\perp} \\ & + \mathbf{M}_{\mathbf{Q}}^{\perp}N_{-\mathbf{Q}} + \mathbf{M}_{-\mathbf{Q}}^{\perp}N_{\mathbf{Q}} + i(\mathbf{M}_{\mathbf{Q}}^{\perp}N_{-\mathbf{Q}} - \mathbf{M}_{-\mathbf{Q}}^{\perp}N_{\mathbf{Q}}) \times \mathbf{P} + i\mathbf{M}_{\mathbf{Q}}^{\perp} \times \mathbf{M}_{-\mathbf{Q}}^{\perp} \end{aligned}$$

Final polarisation

$$\mathbf{P}' = \sigma_{\mathbf{Q}} / \mathbf{P}'\sigma_{\mathbf{Q}}$$

Blume – Maleyev (1963) general theory for polarized neutron scattering

... yields two expressions

$$\mathbf{P} = 0$$

for scattering intensity

$$\sigma_{\mathbf{Q}} = \sigma_{\mathbf{Q},\text{coh}}^{\text{N}} + \sigma_{\mathbf{Q},\text{isotope-inc}}^{\text{N}} + \sigma_{\mathbf{Q},\text{spin-inc}}^{\text{N}} \quad \sigma_{\mathbf{Q},\text{coh}}^{\text{N}} = |N_{\mathbf{Q}}|^2$$

$$+ |M_{\mathbf{Q}}^{\perp}|^2 + \mathbf{P}(N_{-\mathbf{Q}}M_{\mathbf{Q}}^{\perp} + M_{-\mathbf{Q}}^{\perp}N_{\mathbf{Q}}) + i\mathbf{P}(M_{-\mathbf{Q}}^{\perp} \times M_{\mathbf{Q}}^{\perp})$$

magnetic *magnetic-nuclear interference* *chirality*

and final polarized intensity

$$\mathbf{P}'\sigma_{\mathbf{Q}} = \mathbf{P}\sigma_{\mathbf{Q},\text{coh}}^{\text{N}} + \mathbf{P}\sigma_{\mathbf{Q},\text{isotop-inc}}^{\text{N}} - \frac{1}{3}\mathbf{P}\sigma_{\mathbf{Q},\text{spin-inc}}^{\text{N}}$$

$$+ M_{\mathbf{Q}}^{\perp}(\mathbf{P}M_{-\mathbf{Q}}^{\perp}) + M_{-\mathbf{Q}}^{\perp}(\mathbf{P}M_{\mathbf{Q}}^{\perp}) - \mathbf{P}M_{\mathbf{Q}}^{\perp}M_{-\mathbf{Q}}^{\perp}$$

$$+ M_{\mathbf{Q}}^{\perp}N_{-\mathbf{Q}} + M_{-\mathbf{Q}}^{\perp}N_{\mathbf{Q}} + i(M_{\mathbf{Q}}^{\perp}N_{-\mathbf{Q}} - M_{-\mathbf{Q}}^{\perp}N_{\mathbf{Q}}) \times \mathbf{P} + iM_{\mathbf{Q}}^{\perp} \times M_{-\mathbf{Q}}^{\perp}$$

Blume – Maleyev

general theory for polarized neutron scattering

... yields two expressions

for scattering intensity

$$\sigma_{\mathbf{Q}} = \sigma_{\mathbf{Q},\text{coh}}^{\text{N}} + \sigma_{\mathbf{Q},\text{isotope-inc}}^{\text{N}} + \sigma_{\mathbf{Q},\text{spin-inc}}^{\text{N}}$$

$$+ |\mathbf{M}_{\mathbf{Q}}^{\perp}|^2 + \mathbf{P}(\mathbf{N}_{-\mathbf{Q}}\mathbf{M}_{\mathbf{Q}}^{\perp} + \mathbf{M}_{-\mathbf{Q}}^{\perp}\mathbf{N}_{\mathbf{Q}}) + i\mathbf{P}(\mathbf{M}_{-\mathbf{Q}}^{\perp} \times \mathbf{M}_{\mathbf{Q}}^{\perp})$$

magnetic
magnetic-nuclear interference
chirality

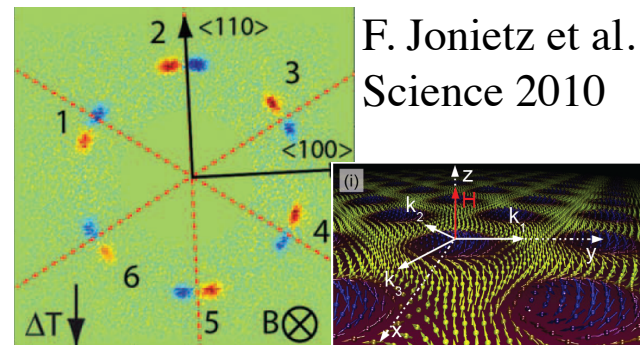
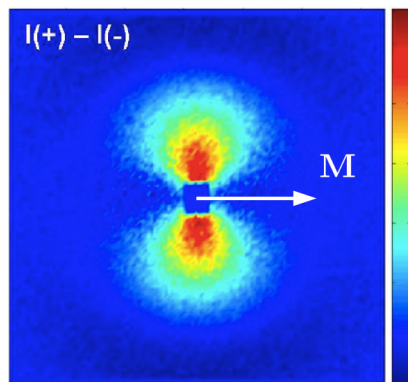
and final polarized intensity

$$\mathbf{P}'\sigma_{\mathbf{Q}} = \mathbf{P}\sigma_{\mathbf{Q},\text{coh}}^{\text{N}} + \mathbf{P}\sigma_{\mathbf{Q},\text{isotop-inc}}^{\text{N}} - \frac{1}{3}\mathbf{P}\sigma_{\mathbf{Q},\text{spin-inc}}^{\text{N}}$$

$$+ \mathbf{M}_{\mathbf{Q}}^{\perp}(\mathbf{P}\mathbf{M}_{-\mathbf{Q}}^{\perp}) + \mathbf{M}_{-\mathbf{Q}}^{\perp}(\mathbf{P}\mathbf{M}_{\mathbf{Q}}^{\perp}) - \mathbf{P}\mathbf{M}_{\mathbf{Q}}^{\perp}\mathbf{M}_{-\mathbf{Q}}^{\perp}$$

$$+ \mathbf{M}_{\mathbf{Q}}^{\perp}\mathbf{N}_{-\mathbf{Q}} + \mathbf{M}_{-\mathbf{Q}}^{\perp}\mathbf{N}_{\mathbf{Q}} + i(\mathbf{M}_{\mathbf{Q}}^{\perp}\mathbf{N}_{-\mathbf{Q}} - \mathbf{M}_{-\mathbf{Q}}^{\perp}\mathbf{N}_{\mathbf{Q}}) \times \mathbf{P} + i\mathbf{M}_{\mathbf{Q}}^{\perp} \times \mathbf{M}_{-\mathbf{Q}}^{\perp}$$

creates P'



F. Jonietz et al.
Science 2010

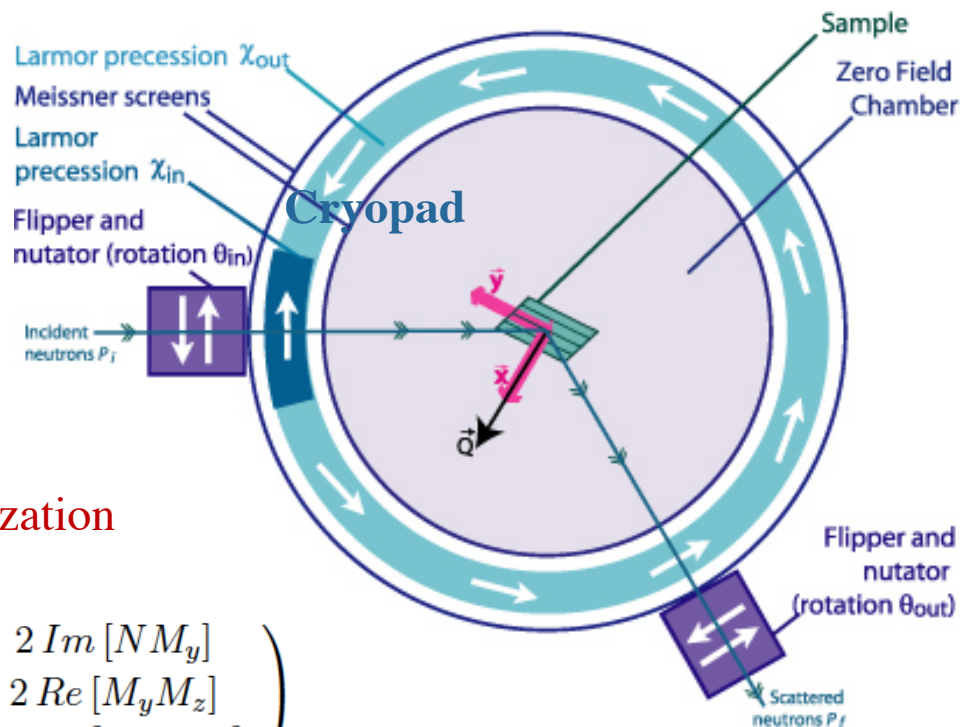
Skyrmions

Polarization reversal

Spherical neutron polarimetry

Jane Brown, Francis Tasset

Cryopad



$$\mathbf{P}'\sigma = (|N|^2 + \mathcal{R})\mathbf{P} + \mathbf{P}''$$

\mathbf{P}'' created polarization

$$\mathcal{R} = \begin{pmatrix} -|M_y|^2 - |M_z|^2 & 2 \operatorname{Im} [NM_z] & 2 \operatorname{Im} [NM_y] \\ -2 \operatorname{Im} [NM_z] & +|M_y|^2 - |M_z|^2 & 2 \operatorname{Re} [M_y M_z] \\ -2 \operatorname{Im} [NM_y] & 2 \operatorname{Re} [M_z M_y] & -|M_y|^2 + |M_z|^2 \end{pmatrix}$$

$$\mathbf{P}'' = (-2 \operatorname{Im} [M_y M_z], 2 \operatorname{Re} [NM_y], 2 \operatorname{Re} [NM_z])$$

Half-polarized experiments – polarization reversal

$$\begin{aligned} \sigma_Q(\mathbf{P}) - \sigma_Q(-\mathbf{P}) &= 2\mathbf{P}(N_{-Q}M_Q^\perp + M_{-Q}^\perp N_Q) + 2i\mathbf{P}(M_{-Q}^\perp \times M_Q^\perp) \\ &= -2 \operatorname{Im} [M_y M_z]_x, 2 \operatorname{Re} [NM_y], 2 \operatorname{Re} [NM_z] \text{ for } \mathbf{P} = P_x, P_y, \text{ and } P_z \end{aligned}$$

Single crystal diffuse magnetic scattering with “XYZ” polarization analysis + polarisation reversal

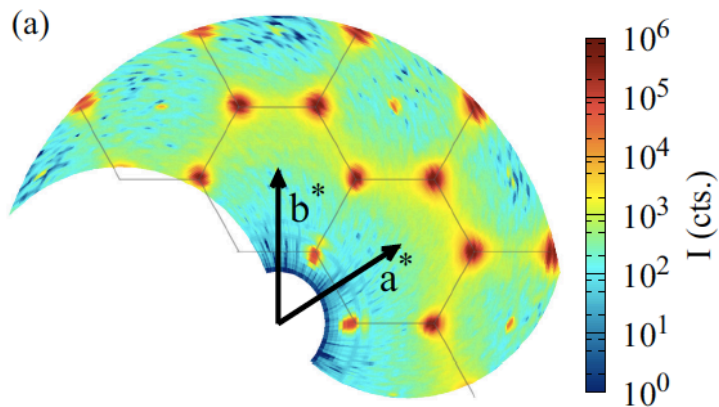
⇒ Separation of all terms in the Blume – Maleyev equations

W. Schweika 2010 J. Phys.: Conf. Ser. **211** 012026

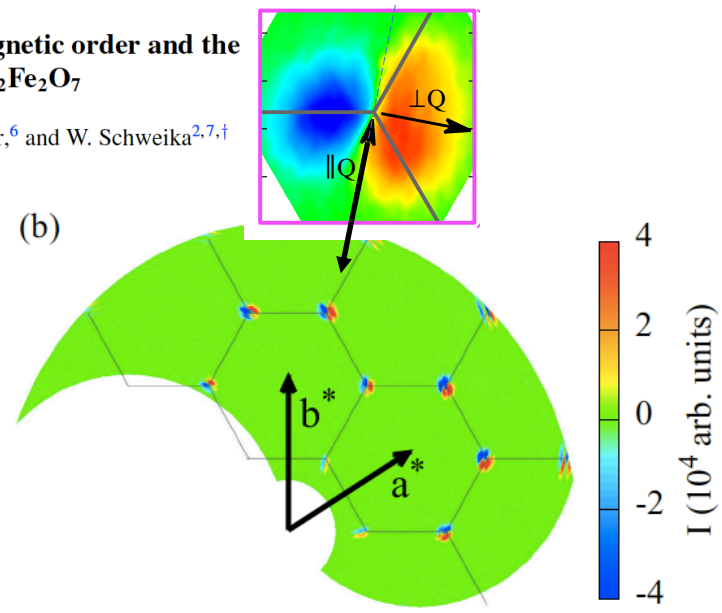
PHYSICAL REVIEW B **97**, 144402 (2018)

Neutron diffraction study and theoretical analysis of the antiferromagnetic order and the scattering in the layered kagome system $\text{CaBaCo}_2\text{Fe}_2\text{O}_7$

J. D. Reim,^{1,2,*} E. Rosén,² O. Zaharko,³ M. Mostovoy,⁴ J. Robert,⁵ M. Valldor,⁶ and W. Schweika^{2,7,†}

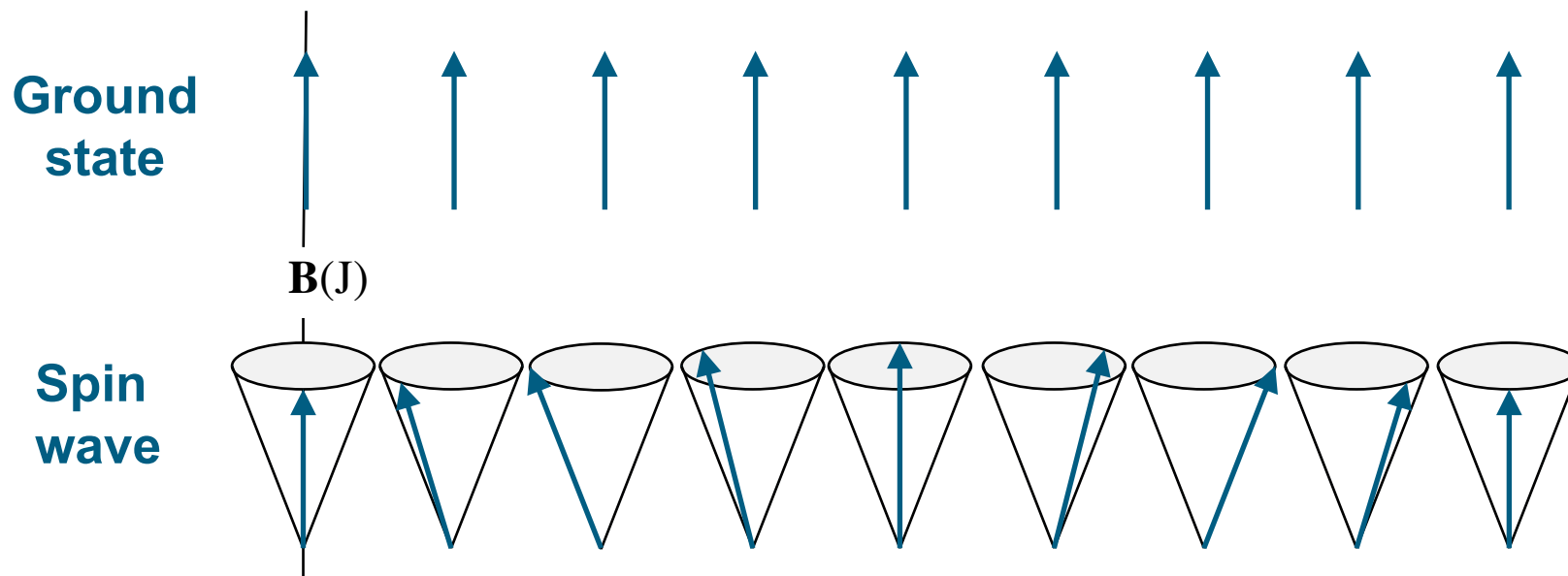


total magnetic scattering

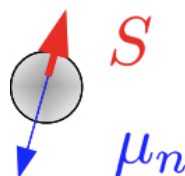


Interference due to chirality

Spin wave excitations



Spin waves, aren't they chiral?
left or right handedness?
is the picture correct?
helix or cycloid?



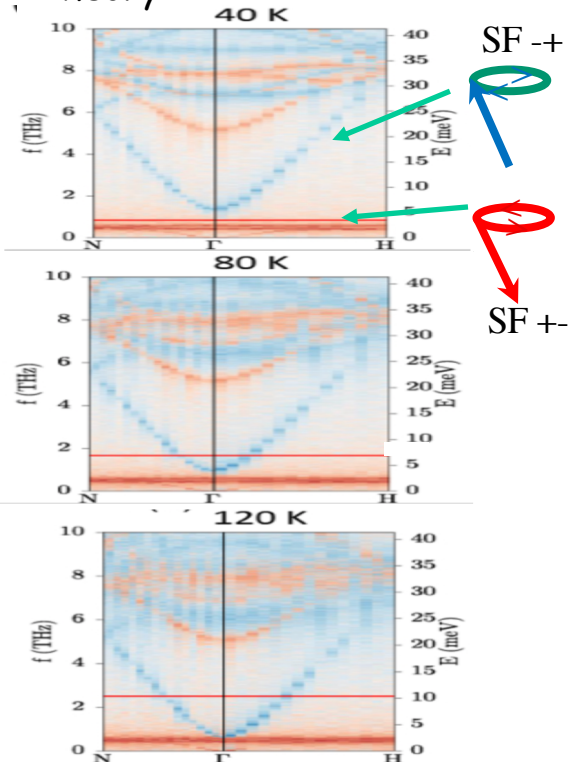
and what about neutrons
aren't they chiral, too?

The Spin Seebeck effect

is caused by thermally excited spin dynamics that are converted to a voltage by the inverse spin Hall effect at the interface to a heavy metal contact.

Neutron Scattering Results

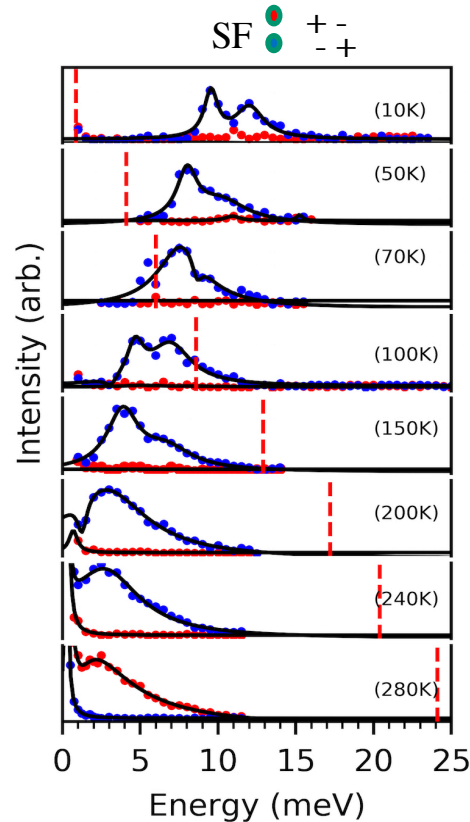
Theory



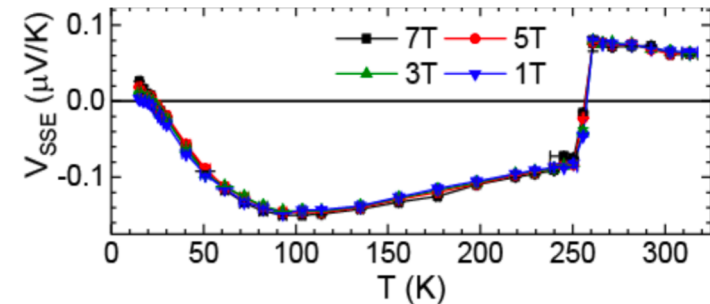
Landau-Lifshitz-Gilbert

Nature Comms. 7 10452 (2016)

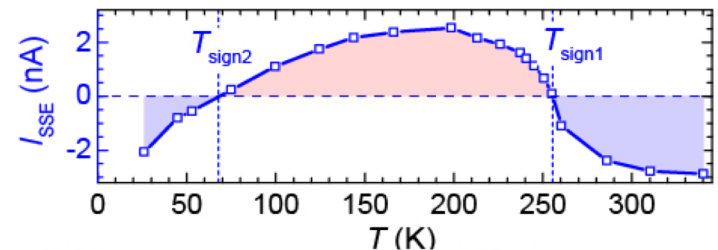
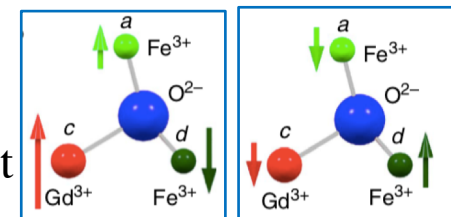
magnons are chiral, of course ...



Tb-iron garnet



Gd-iron garnet



courtesy of Dan Mannix, ESS

Magnetization and spin densities

wide angle Bragg diffraction for atomic resolution

I^+ (\mathbf{P} parallel \mathbf{H}) and I^- (\mathbf{P} antiparallel \mathbf{H})

“flipping ratio” measurements sensitive to small \mathbf{M}

$$R = \frac{I^+}{I^-} = \frac{N_{\mathbf{Q}}N_{-\mathbf{Q}} + (N_{\mathbf{Q}}\mathbf{M}_{-\mathbf{Q}}^{\perp} + N_{-\mathbf{Q}}\mathbf{M}_{\mathbf{Q}}^{\perp}) + \mathbf{M}_{\mathbf{Q}}^{\perp}\mathbf{M}_{-\mathbf{Q}}^{\perp}}{N_{\mathbf{Q}}N_{-\mathbf{Q}} - (N_{\mathbf{Q}}\mathbf{M}_{-\mathbf{Q}}^{\perp} + N_{-\mathbf{Q}}\mathbf{M}_{\mathbf{Q}}^{\perp}) + \mathbf{M}_{\mathbf{Q}}^{\perp}\mathbf{M}_{-\mathbf{Q}}^{\perp}}.$$

example $M_z = 0.1N$

$$I = 1.01|N|^2, \quad \text{and} \quad \frac{I^+}{I^-} = \frac{(1 + 0.1)N^2}{(1 - 0.1)N^2} = \frac{1.21}{0.81} \approx 1.5$$

solve nuclear structure first

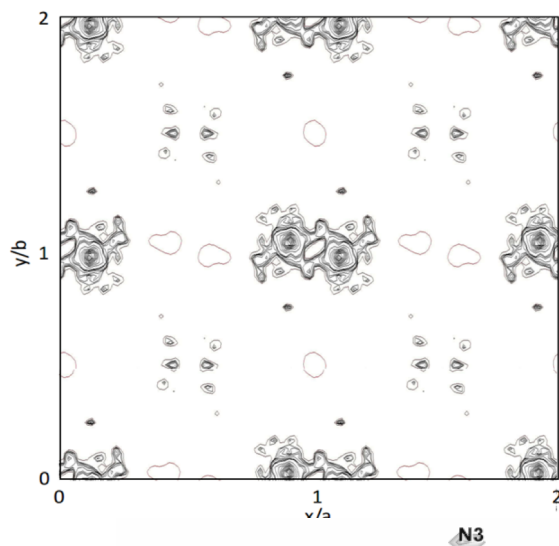
– collecting Bragg peaks from single crystal samples

D3 ILL Grenoble, Navid Qureshi

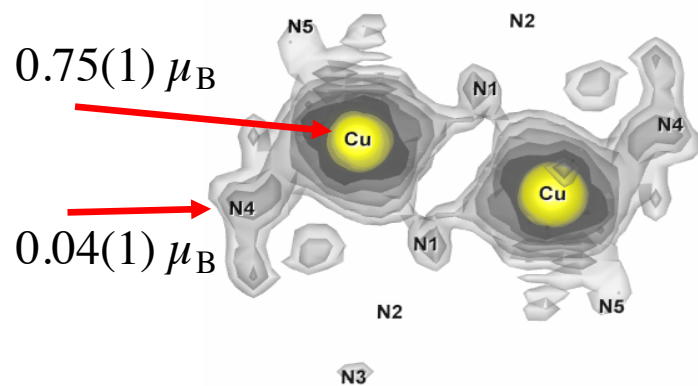
now developed for 2D powder diffraction

Examples of using nuclear magnetic interference (no polarization analysis)

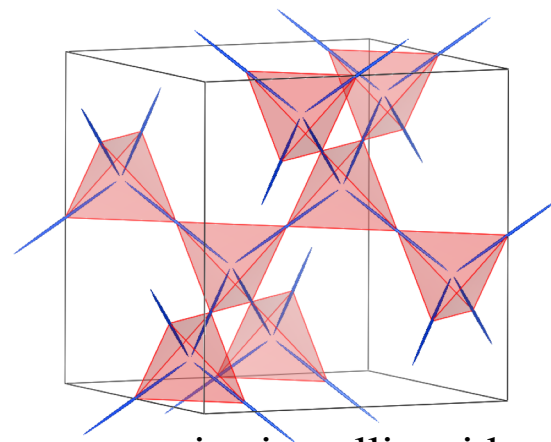
Spin densities in molecular magnets



*Aebersold
JACS 1998*



Local susceptibilities anisotropies



magnetisation ellipsoids on Ho
(axis along $\langle 111 \rangle$)

spin-ice
Ho₂Ti₂O₇

at 5K 1T:
 $\chi_{\parallel} = 11.3(1) \mu_B$
 $\chi_{\perp} = -0.1(1) \mu_B$

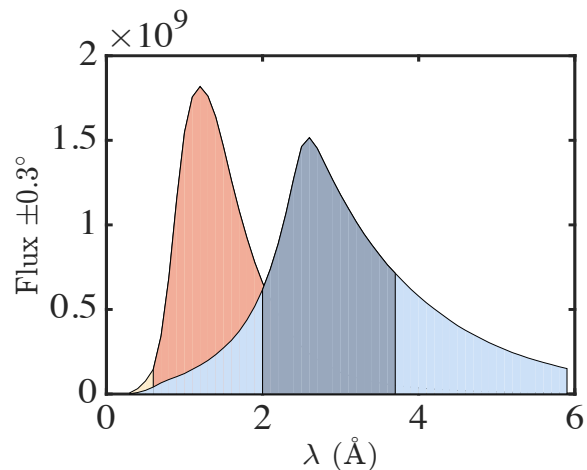
*Iurii Kibalin & Arsen Gukasov
PRB 2019 (from powder)*

Outline

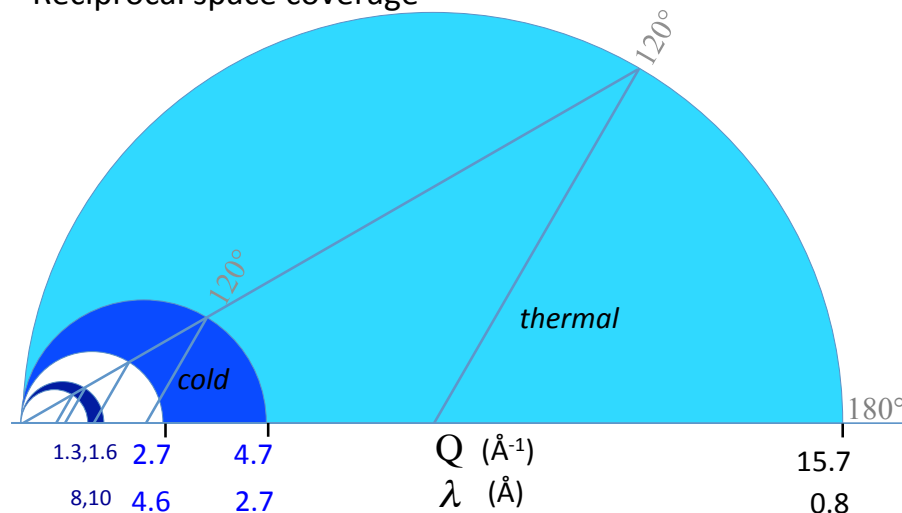
- **Neutron spins in magnetic fields**
 - experimental devices => instruments
- **Scattering and Polarization**
 - spin-dependent nuclear interaction
 - magnetic interaction
- **Blume-Maleyev Equations**
 - examples
- **outlook for ESS**

ESS polarised single crystal instrument MAGiC

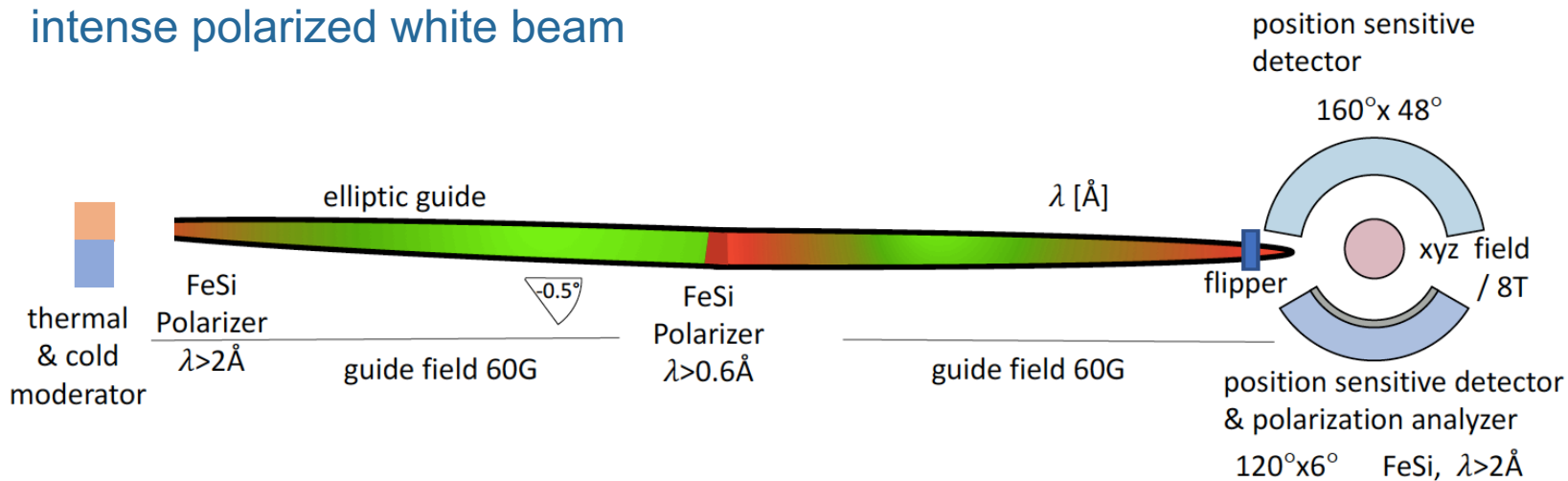
in construction LLB, JCMS, PSI
User operation 2023



Neutron time-of-flight Laue instrument
Reciprocal space coverage



intense polarized white beam



virtual MAGIC experiments

Cases

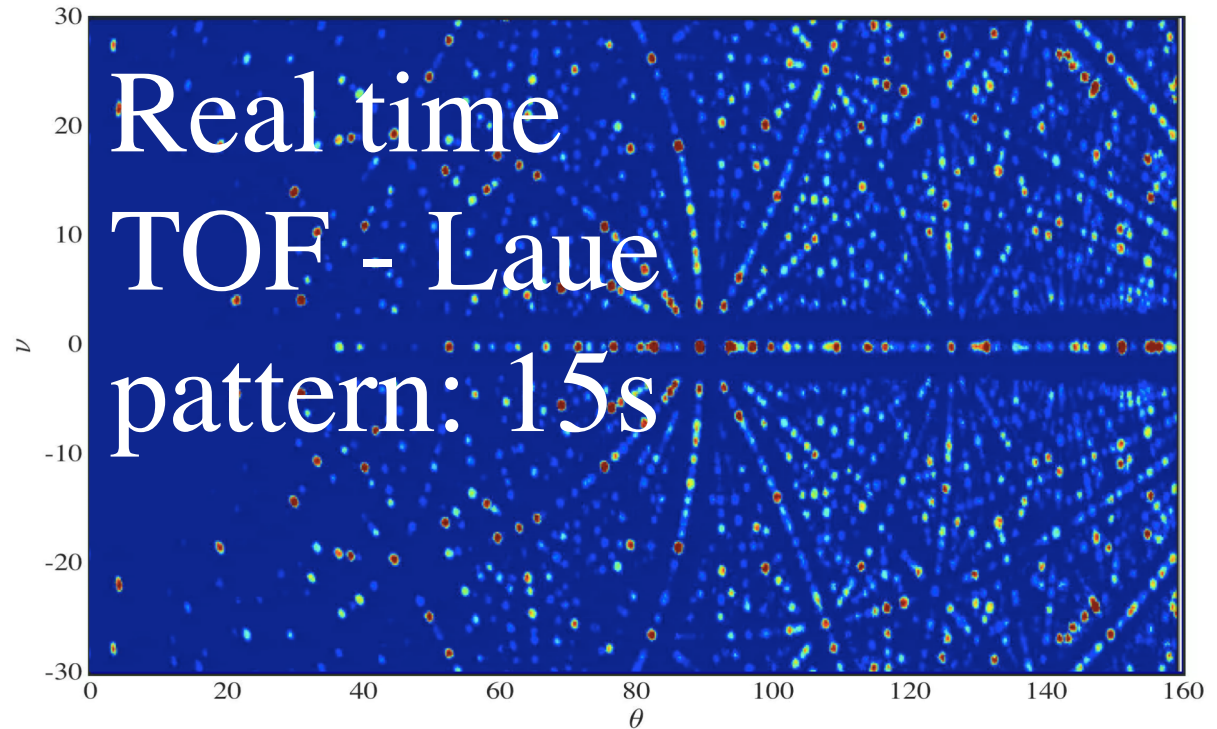
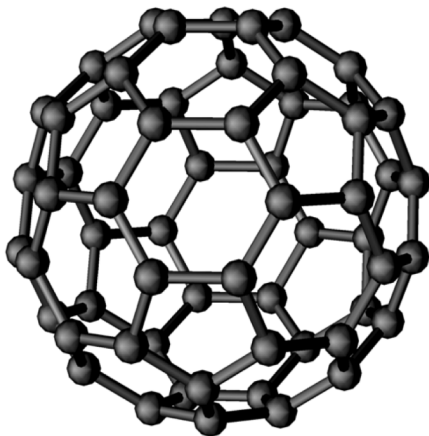
HoMnO₃

BiFeO₃

Spin ice

Bucky ball

Molecular magnets



C₆₀: a=14 Å, 1 mm³ sample

Thermal spectrum @ full pulse length

Full data collection: 1mm³ ~ minute

virtual MAGIC experiments

Cases

HoMnO₃

BiFeO₃

Spin ice

Bucky ball

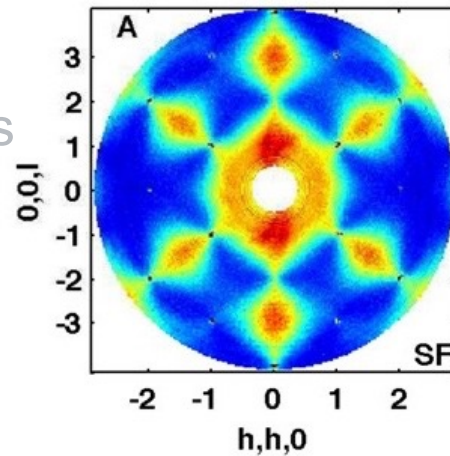
Molecular magnets

Ho₂Ti₂O₇

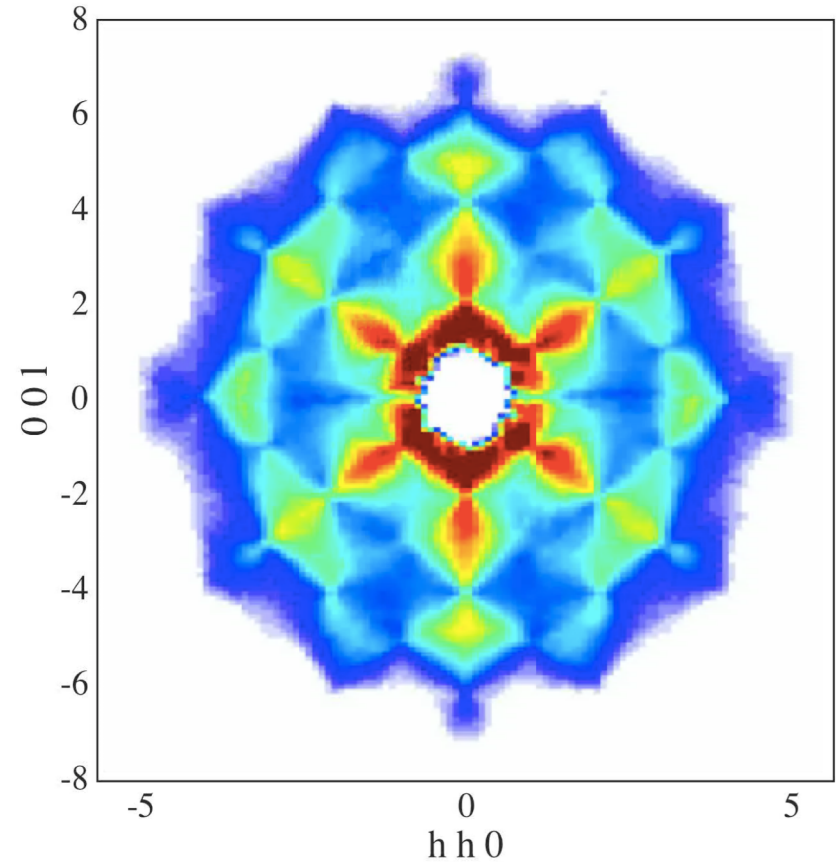
T. Fennell *et al.*

Science 2009

D7: 2×10^6 n/s/cm²



MAGiC: 2×10^9 n/s/cm²



10 min & 10 mm³

virtual MAGIC experiments

Cases

HoMnO₃

BiFeO₃

Spin ice

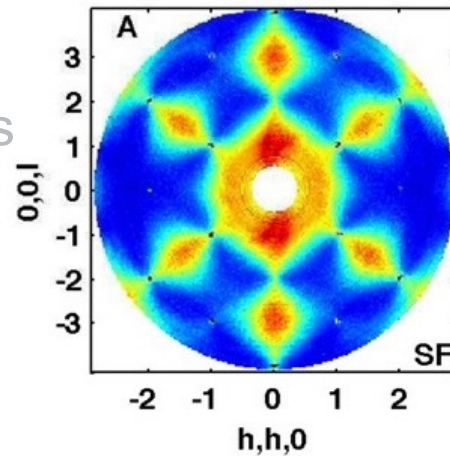
Bucky ball

Molecular magnets

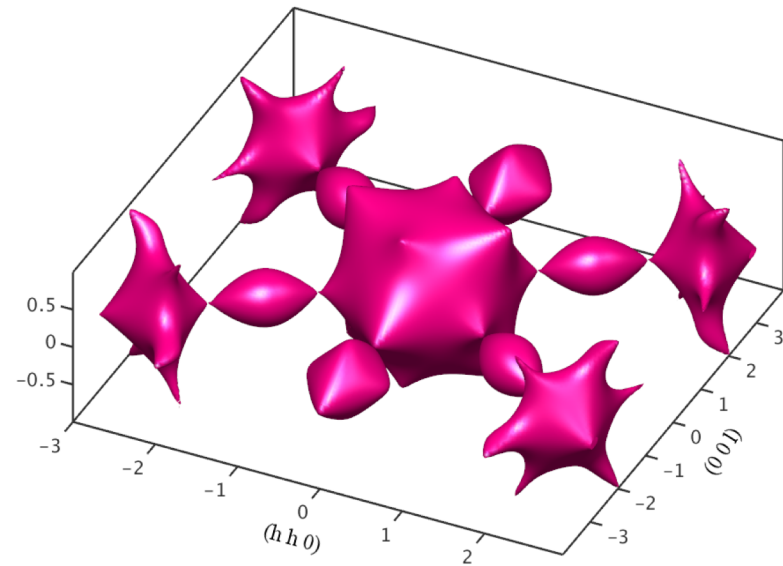
Ho₂Ti₂O₇

T. Fennell *et al.*
Science 2009

D7: 2×10^6 n/s/cm²



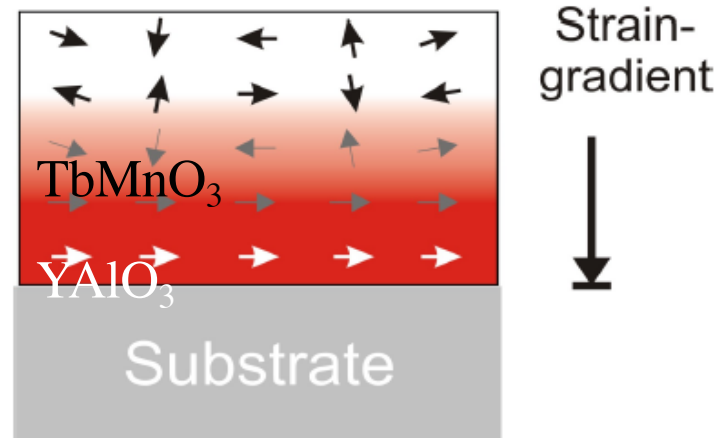
MAGiC: 2×10^9 n/s/cm²



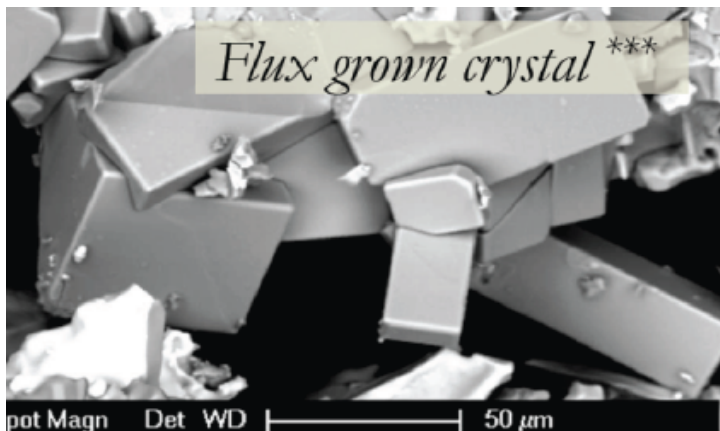
10 min & 10 mm³

Future: Small moments, small samples, or heterostructures

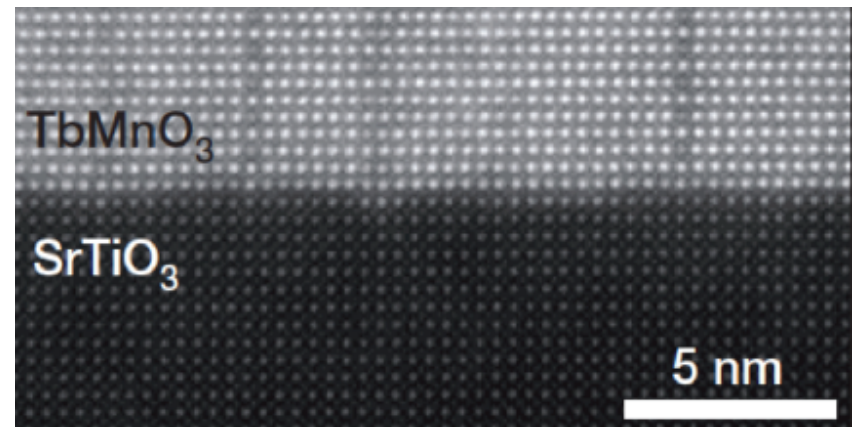
Many single crystalline materials are only available in very small quantities



Adapted from J. White et al., Phys. Rev. Lett. **111**, 037201 (2013)



Courtesy Dr. M. Valldor



S. Farokhipoor et al, Nature Materials **515** , 379 (2015)

Summary

Neutron Spin

S

nuclear interaction

N coherent
 $I \neq 0$ spin-incoherent
 isotropic

nsf
 1/3 nsf
 2/3 sf

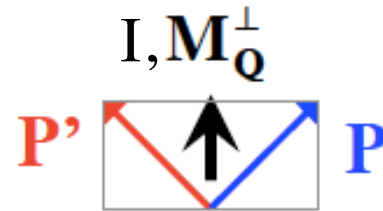
hydrogenous materials

magnetic moment

μ

magnetic dipolar interaction

M_Q^\perp anisotropic



complex frustrated magnetism

Polarisation

$$2\langle \mathbf{S} \rangle = \mathbf{P}$$

probing vector properties and interference
 separation of scattering terms